CALIBRATION OF MAGNETIC DETECTORS ON MAST-U

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MAST-Upgrade

- MAST-U is a Major upgrade of the Mega Ampere Spherical Tokamak, (MAST).

- Three main objectives:
  - Adding to the knowledge base for ITER,
  - Exploring the case for a ST-based Component Test facility.
Magnetic diagnostic system

- MAST-U has 25 Poloidal field coils (enclosed in metal cases).
- 398 discrete pickup coils.
- 102 flux loops connected separately and differentially.
- 91 partial flux loops (saddle loops).
- 56 Rogowskii coils
- 70 Mirnov coils.
Purpose of magnetic diagnostics

Magnetic diagnostics measure: **magnetic fields**, **active currents** and **passive currents** for:

- Plasma control
- Determining magnetic equilibrium for physics analysis
- Measuring magnetic fluctuations.
The requirements

❖ Typical requirements for the magnetic equilibrium:
  • Separatrix and pedestal spatial positions to <1cm.
  • Core q-profile values < 0.1
  • Pressure <10%

❖ Experience on existing tokamaks shows that these requirements are difficult to achieve.
The challenges

- Large number of magnetic diagnostics.
- Uncertainties in bench calibrations.
- Uncertainties in diagnostic and pf coil placement.
- Possible parasitic pickups.
- Issues to do with electronic hardware.
- Mistakes in signal polarity.
- Asymmetries in vessel, pf coils, flux loops.
Standard calibration technique

A detailed account of the magnetic calibration on TCV is described by Moret et al.¹

❖ Discrepancies in flux loops \( \lesssim 2\% \) and magnetic pickups \( \lesssim 5\% \) using “prior” information about diagnostic and machine geometry

❖ Significant reduction in discrepancies achieved by adjusting the diagnostic and pf coil geometry and scaling factors.

✓ Demonstrates that an analysis using the complete magnetic data set can substantially improve calibration.

✗ Method used was specific to calibration problem.

✗ Difficult for method to take account of other factors.

✗ Difficulties in calibrating (3D) partial flux loops.

¹Moret et al: Review of Scientific Instruments 69, 2333 (1998);
What are the Ideal features of a calibration method?

✓ Assimilates data from multiple discharges.
✓ Assimilates data from heterogeneous magnetic detectors.
✓ Incorporates flexible and adaptable models, able to take account parasitic pickups, distortions in pf coils, 3D effects etc.
✓ Method is mathematically robust and able to encode prior information.
The Minerva framework

✓ Framework for setting up and solving Bayesian Inference problems.
✓ Modular and specifically designed for large-scale heterogeneous problems (eg with >1000 free parameters).
✓ Contains many existing models of Fusion-relevant diagnostics.
Bayes formula

If $H$ are a set of free parameters, and $D$ the data set, Bayes’ formula can be written:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

- posterior
- likelihood
- evidence
- prior

Bayes' formula

evidence

posterior
The Minerva approach

• Minerva implements a *declarative model* containing all the components of the joint distribution $P(H, D)$.

• An accompanying programme operates on the model to solve a particular problem of interest.

• Methods within Minerva permit exploration of the model including:
  • Adjustment of model parameters,
  • Computation of the Maximum a posteriori (MAP)
  • Sampling from the posterior distribution $P(H|D)$.
  • Linear approximations.
Diagnostic Model

- **Pickup coils**: treated as point measurements.
  - free parameters: $R_p$, $Z_p$, $\theta_p$ (poloidal orientation), $S_\theta$ (scaling factor).
- **Prior distributions**:  
  - Uncorrelated Gaussian distributions based on the best-available information (eg. machine drawings or calibrations).
The Minerva model is described using a generalised Bayesian Graph showing:

- Components of the joint distribution
  \[ P(\text{free parameters, observations}) \]
- Connected nodes for computing pdf parameters.

Bayesian Graph contains a full description of model including all assumptions and regularisations.

Graphical models tend to be very large and unreadable!!

Example calibration model for pickup coils and data from one pf dry-run.
Graphical model organized into subgraphs

- Solution is to organise the Minerva graphs into subgraphs
- Example for calibration of pickups with $n_{pf}$ dry-run datasets

Graphical model organized into subgraphs:

- Sub-graphs for free parameters
- Sub-graphs for data set 1, 2, ..., $n$

Free parameters
Graphical model organized into subgraphs

- Solution is to organise the Minerva graphs into subgraphs
- Example for calibration of pickups with $n_{pf}$ dry-run datasets

Sub-graph for free parameters

Sub-graph for data set $n$

Likelihood

Sub-graph for data set 1

Sub-graph for data set 2

Sub-graph for data set $n$
Practicalities

- No MAST-U data, so use data from MAST as an example.
- Use 20 dry runs (no-plasma pulses); average data over flat-top time.
- Configuration defined using “generic” EFIT++ XML files readable by Minerva.
- Use MAP solution to initialize Markov-chain Monte Carlo (MCMC) algorithm.
Results of MCMC sampling for magnetic probes.

+/- 1σ about mean of MCMC samples

\[ \delta R_p \text{ [cm]} \]

\[ \delta Z_p \text{ [cm]} \]

\[ \delta \theta_p \text{ [deg]} \]

\[ \delta S_p \]
Correlations between free parameters

- Sampling from $P(H|D)$ provides
  - Uncertainties ("error bars") on individual parameters.
  - Evidence of correlations between parameters.
MCMC samples from the posterior (single probe)

- Each point in graph is a single sample.
- Colouring is logPDF value: points have higher density close to MAP (in red).
Using the calibration parameters

Possible ways of using the calibration data for equilibrium determination:

I. Compute uncertainties on inferred parameters from marginal posterior distribution (i.e. ignoring correlations in subsequent analysis)

II. Obtain tabulated samples of calibration parameters

II. Use these values to run equilibrium code (EFIT++) multiple times.

I. Include $P(H|D_{\text{callib}})$ as a prior distribution in the equilibrium Bayesian inference problem, and marginalise over $H$. 
EFIT++ reconstructions with uncertainties

- EFIT++ code run on MAST discharge 24459 (t=288ms) using 500 calibration sets.
- Separatrix has largest uncertainty in proximity of the x-points.
  - \((R, Z)\) coordinates at the separatrix are correlated
  - Marginal distribution exhibit non-symmetric distributions.
The next steps

- **Pickup coils:**
  - free parameters: $R_p, Z_p, \theta_p$ (poloidal orientation), $S_\theta$ (scaling factor), $\phi_p$ (toroidal angle).

- **Flux loops:** treated as a circular axisymmetric loops.
  - free parameters: $R_f, Z_f, S_f$ (scaling factor).

- **Saddle loops:** each one treated as the difference of two flux loops:
  - free parameters: $R_{inner}, Z_{inner}, R_{outer}, Z_{outer}, S_s$ (scaling factor).

- **Pf coils:**
  - free parameters (2-D): $\delta R, \delta Z, S_{pf}$ (scaling factor).
  - free parameters (3-D): $\delta X, \delta Y, \delta Z, \delta X_{rot}, \delta Y_{rot}, \delta Z_{rot}$. 
Bayesian Inference of full Equilibrium problem

Minerva graphical model of the tokamak equilibrium problem.

❖ Contains multiply nested Bayesian sub-graphs
❖ Includes multiple diagnostic systems
❖ Grad-Shafranov constraint
❖ Generic model applicable to multiple tokamaks
Conclusions

❖ Bayesian Inference model using Minerva framework:
  • Provides a Mathematically robust method to solve the calibration problem.
  • Minerva framework provides flexibility for testing different models, including treating tokamak as a full 3-D device.
  • Model assumptions are clear.
  • Model results on inferred parameters provides uncertainties including all correlations.

❖ Results from the calibration:
  • Used in EFIT++ to provide uncertainties on equilibrium-parameters.
  • Can provide a prior distribution in the (full) equilibrium Bayesian inference problem.