Two- and Three-Dimensional Tomography of Radiated Power using Imaging Bolometers in Toroidal Devices

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Outline

- IRVB concept
- Tomography
- 3D tomography in LHD (R. Sano)
  - Bolometer channels and plasma grid definition
  - Initial numerical test with standard linear solver
  - Extension of linear system with prior information
  - 3D reference profile function and iterative optimizer
  - Numerical test and application to experimental data
- 2D tomography on KSTAR (J. H. Jang)
  - KSTAR IRVB setup
  - Tomography technique
  - Phantom reconstruction tests
  - Experimental results
Imaging Bolometer (IRVB) Concept

Calculate $P_{rad}$ from foil $T$ using 2D heat diffusion equation

\[ \Omega_{rad} + \Omega_{bb} = \frac{1}{\kappa} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \]

2-D Laplacian

\[ \Omega_{bb} = \frac{\varepsilon \sigma_s B (T^4 - T_0^4)}{kt_f} \quad \varepsilon \approx 1 \]

black body cooling term

\[ \Omega_{rad} = \frac{P_{rad}}{kt_f l^2} \]

foil thermal diffusivity

foil thermal conductivity

foil thickness

bolometer pixel area

Plasma radiation absorbed by foil

IR measured by camera

Thin foil

copper frame

Thermal diffusion to frame

Tomography problem: line averaged \[ \rightarrow \] local information

Plasma parameter distribution (unknown)

Line of Sight Integration (forward problem)

Tomography (Reconstruction) (inverse problem)

Line-integrated data (measurement output)

\[ \mathbf{P}_i : \text{Line-integrated detector Field of View (FoV) data} \]

\[ \mathbf{H}_{ij} : \text{Projection matrix (i detectors x j plasma voxels)} \]

\[ \mathbf{S}_j : 1D, 2D \text{ or } 3D \text{ Plasma parameter distribution grid} \]

Projection matrix, \[ \mathbf{H}_{ij} = V_{ij} \Omega_{ij} / 4\pi \]

\[ V \text{ - intersecting volume of FoV } i \text{ and S grid } j \]

\[ \Omega_{ij} = A_i / l_{ij}^2 \text{ - detector area, } A_i, \text{ solid angle, } \Omega_{ij} \]

⇒ Relation between \( \mathbf{P} - \mathbf{S} \)

⇒ Must be inverted to get \( \mathbf{S} \) from \( \mathbf{P} \)
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IRVBs and plasma voxels are designed as 3D tomography system

Diagnostics (IRVB × 4)
IRVB channels: 1008+1008+560+620=3,196ch

Plasma voxel (grid element) (cylindrical coordinate)
Horizontal (R):
- 5 cm
- 54 divisions (2.4 m < R < 5.1 m)
Vertical (Z):
- 5 cm
- 52 divisions (-1.3 m < Z < 1.3 m)
Toroidal (φ):
- 1 degree
- 360 divisions
Total number of voxels: 1,010,880

ROI (Region of Interest) voxel (tomography target):
masking process, helical periodicity (n=10)
⇒ 16,188 voxels (0° < φ < 18°)

P = HS
P (3196 ch)
S (16188 voxels)
H (3196 × 16188)
3D reconstruction is achieved by standard linear solver for tomography. → But, large number of artifacts should be reduced.

**Numerical test**

Model (EMC3-EIRENE), $S$

![Model images](image)

Synthetic image, $P$

![Synthetic image](image)

Reconstructed, $\hat{S}$ (with 10% noise)

![Reconstructed images](image)

Reconstructed $\hat{P}$

![Reconstructed image](image)
Standard linear solver is extended by prior information

Standard linear solver (Tikhonov regularization)

Lagrange function

$$\Lambda(S) = \gamma \|CS\|^2 + \frac{||HS - P||^2}{M}$$

- $H$: Projection matrix
- $P$: Integrated data
- $S$: Plasma parameter distribution
- $M$: Total number of diagnostics channels
- $\gamma$: Regularization parameter
- $C$: Identity matrix

Extended solver (with prior information)

Lagrange function

$$\Lambda(S) = \gamma \|C(S - \alpha m)\|^2 + \frac{||HS - P||^2}{M}$$

- $m$: Reference profile
- $\alpha$: Weighting factor

Euclidean distance between reference profile and reconstructed profile

Flow of 3D tomography with prior information

Reference profile is processed as a rough estimation by diagnostics data and knowledge of plasma.

IRVB images $\mathbf{P}$

Reference processing (fitting)

3D reference profile, $\mathbf{m}$

Reconstruction (extended)

Reconstruction (standard)

Prior information

Weighting factor $\alpha$ is fixed to 0.5 (by numerical test results)
3D model function is employed for \( \mathbf{m} \) (Reference profile)

Model

\[
PP(r, z, \phi) = \frac{\{(r - R_{\text{center}}) \cos(\phi) - (z \sin(\phi))\}^2}{b^2} + \frac{\{(r - R_{\text{center}}) \sin(\phi) + (z \cos(\phi))\}^2}{a^2}
\]

Definition of ellipse

\((1 - w)^2 \leq PP(r, z, \phi) \leq 1^2\)

\[
S_{\text{model}}(r, z, \phi) = c \left(1 + \gamma_{\text{in-out}} \cos \left(\frac{(R - R_{\text{center}})}{2(R_{\text{center}} - R_{\text{edge}})\pi}\right)\right) \left(1 + \gamma_{\text{peak}} XX(r, z, \phi)\right) \exp \left(-\frac{(PP - (1 - w/2)^2)^2}{(w/2)^2}\right)
\]

Asymmetry part

X-point peaking part

\[
XX(r, z, \phi) = \exp \left(-\frac{\{(R - R_{\text{center}}) \cos(\phi) - (z \pm a(1 - d_x w)) \sin(\phi)\}^2}{2(d_x w a)^2}\right)
\]

\(a\): semi major radius
\(b\): semi minor radius
\(w\): width of radiation region (ratio)
\(R_{\text{center}}\): center of radiation region
\(c\): radiation intensity
\(\gamma_{\text{in-out}}\): asymmetric factor (inboard – outboard)
\(\gamma_{\text{peak}}\): Magnitude of local peak
\(d_x\): Location of local peak

8 free parameters
Reference profile is processed to fit IRVB image to model projection

Initial parameters

\[(b_0, a_0, w_0, \gamma_{i-o0}, \gamma_{p0}, R_0, c_0, d_{x0})\]

\[\text{b-a scan (n x n)}\]

\[(b_1, a_1, w_0, \gamma_{i-o0}, \gamma_{p0}, R_0, c_0, d_{x0})\]

\[\text{w-} \gamma_{a} \text{ scan (n x n)}\]

\[(b_0, a_0, w_1, \gamma_{i-o1}, \gamma_{p0}, R_0, c_0, d_{x0})\]

\[\text{\(\gamma_p - d_X\) scan (n x n)}\]

\[(b_0, a_0, w_0, \gamma_{i-o0}, \gamma_{p1}, R_0, c_0, d_{x1})\]

\[\text{R scan(n)}\]

\[(b_0, a_0, w_0, \gamma_{i-o0}, \gamma_{p0}, R_1, c_0, d_{x0})\]

\[\text{c scan (n)}\]

\[(b_0, a_0, w_0, \gamma_{i-o0}, \gamma_{p0}, R_0, c_1, d_{x0})\]

Normalized experimental Image (IRVB data) \[\textbf{P}\]

Normalized model projection (Model) \[\textbf{H}_m\]

Mean square error, \(\varepsilon_n^2\), (normalized)

Iteration until convergence

1st 2nd 3rd

5th 7th 9th

Number of calculation for 1 iteration = \(3n^2 + 2n\) \((n = 5)\)

Parameter scanning is individually carried out for each IRVB (U, L)
Characteristics of source is recovered (numerical test)

3D source profile (EMC3-EIRENE)

\[ \phi = 0.5^\circ \]
\[ \phi = 9.5^\circ \]

Inboard

Outboard

\[ \delta^2 = \frac{\|\hat{S} - S_0\|^2}{\|S_0\|^2} \]

Reconstructed profile \( \delta^2 = 0.717 \)

Reconstructed profile \( \delta^2 = 0.674 \)

Incident radiation power (W)

Radiation power density (W/cm\(^3\))

\[ \phi = 0.5^\circ \]
\[ \phi = 9.5^\circ \]

\[ \delta^2 \text{ (reconstruction error)} \]

\[ \delta^2 \text{ (reconstruction error)} \]

Magnetic axis

Far X-point

Near X-point

\[ \delta^2 \text{ is reduced by prior information} \]
Experimental reconstruction becomes easily understandable by prior information (artifacts are suppressed)

Standard solver (Tikhonov)
- **Edge radiation**
- **Core radiation (Radiation collapse)**

Extended solver (with prior)
- **Edge radiation**
- **Core radiation (Radiation collapse)**

<table>
<thead>
<tr>
<th>Negative values</th>
<th>Number</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard solver</td>
<td>3299 VOXELS</td>
<td>-111.00</td>
</tr>
<tr>
<td>Extended solver</td>
<td>2731 VOXELS</td>
<td>-60.46</td>
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<tr>
<td>Standard solver</td>
<td>2684 VOXELS</td>
<td>-52.74</td>
</tr>
<tr>
<td>Extended solver</td>
<td>2493 VOXELS</td>
<td>-39.14</td>
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KSTAR IRVB setup

IR camera: FLIR SC7600
- NETD: < 25mK
- Frame rate: 105 Hz
- Resolution: 512 x 640 pixels

Foil: Pt
- Size: 0.002 x 70 x 90 mm
- Double side carbon coating
- Photon energy range: $E_{ph} < 7.5$ keV

Bolometer
- Time resolution: 10 ms
- Aperture: 3.5 mm x 3.5 mm
- 24(tor) x 32 (pol) = 384 ch

**Tomography**: A non-invasive imaging tool for observing the inner structure of the plasmas
- Line-integrated raw data → **tomography is essential**

**Reconstruction**
**Phillips-Thikhonov method**
- Minimizing $J = \text{mean squared error} + \text{signal variation}$
  \[ J = \frac{|f - W \cdot g|^2}{M} + \gamma |L \cdot g|^2 \]
- P-T solution ($\frac{\partial J}{\partial g_i} = 0$)
  \[ g(\gamma) = (W^T \cdot W + M\gamma L^T \cdot L)^{-1} \cdot W^T \cdot f \]
  - $g$ : Reconstructed image
  - $\gamma$ : Optimal regularization parameter
  - $L$ : Laplacian
- **GCV statistics** : optimized $\gamma$
  → **accuracy vs smoothness**
  → high $\gamma$ : smooth, inaccurate
  → low $\gamma$ : accurate, unstable to noise

**Ill-posed problem**
\[ f = W \cdot g \]
- $f$ : Line-integrated image
- $W$ : Weight matrix
- $g$ : Local emission profile (2-D)
KSTAR IRVB Tomography setup

✓ Reconstruction grid: \(1.2 < R < 2.4 \text{ m}, \ -1.5 < Z < 1.5 \text{ m}\)
  \(\rightarrow\) Divided by 63 x 150 plasma pixels (2 cm x 2 cm for each)

✓ First wall geometry of KSTAR is applied in tomography code

KSTAR IRVB line of sight
(description for plasma pixel and bolometer pixel)
Phantom reconstruction tests (1)

✓ Accuracy of KSTAR IRVB tomography is validated by reconstruction of various synthetic images (phantoms)

✓ Total radiated power

\[ P_{rad} = A_{pixel} \times 2\pi \sum R \cdot \epsilon(R,Z) \]

( \( \epsilon(R,Z) \): emissivity at \((R,Z)\) )

✓ Reconstruction error

\[ e_{recon} (%) = \left| \frac{\epsilon_{phantom} - \epsilon_{recon}}{\epsilon_{phantom}} \right| \times 100 \]

More complicated phantoms\[^1\] can also be reconstructed well (Above: inter-ELM radiation pattern, below: during ELM)

\[^1\] J. Jang et al, 2018 Curr. Appl. Phys, 18 461
Phantom reconstruction tests (2)

- High reconstruction accuracy near first wall in KSTAR
  → useful for impurity seeding exp. or plasma-divertor detachment

- Phantom reconstruction tests
  - a) D-shape + Hot spots in
  - b) X-point,
  - c) inboard and
  - d) outboard divertor *
  (10% noise added to line-integrated signal)

Spatial resolution of IRVB tomography

✓ Spatial resolution of IRVB tomography

~ 9 cm

: Two gaussian peak with 9 cm gap can be distinguished
(10% noise added to line-integrated signal)
Exp1) ELM mitigation by Kr seeding

- 1.7x10^{19} Kr particles injected
- ELM suppression (~5\tau_E)
- ELM mitigation (~10\tau_E to the end of shot)
  - 50% reduction in \Delta W_{ELM} \times f_{ELM}

IRVB plays crucial role in impurity study in KSTAR
Exp2) ELM suppression by Kr seeding

- 3.5x10^{19} Kr particles injected
- ELM suppression (~4 \tau_E)
- H-L transition

Kr density can be estimated from Kr radiation (D.E. Post et al. 1977)

$P_{rad}(\rho) = n_e(\rho) n_Z(\rho) L_z(T_e(\rho))$

$L_z$ : radiative cooling rate (coronal equilibrium)

IRVB plays crucial role in impurity study in KSTAR
Exp 3) ITB formation by Kr seeding

- Total 5.2x10^{19} Kr particles injected
- ELM mitigation → H-L back transition → ITB formation
- $T_i$ and $T_e$ profiles show strong core peaking
- Relation between Kr and ITB formation is still under investigation
Conclusions

• IRVBs can be applied to toroidal devices for the purpose of 2D and 3D tomography of radiated power
• In helical devices 3D tomography is desirable, but difficult
  • Improvements can be made using prior information
• In a tokamak with a tangential view
  • 2D profiles can be obtained with good spatial resolution in core and divertor by assuming toroidal symmetry
  • Also total power estimates can be provided.
  • In KSTAR IRVB is the only bolometer and is providing both 2D profiles and total power estimates.

Future work

• 2D tomography of radiation in helical devices using IRVBs assuming radiation is constant on a field line
  • Should be applicable to both LHD and W7-X
Noise and signal can be estimated

\[
S_{IRVB} = \eta_{IRVB} N_{bol} = \frac{\sqrt{10k t_f \sigma_{IR}}}{A_f} \sqrt{\frac{N_{bol}^3 f_{bol}^3}{A_f^2 5\kappa^2}}
\]

Foil properties (Pt):
- \(k = 0.716 \text{ W/cmK}\) – foil thermal cond.
- \(\kappa = 0.2506 \text{ cm}^2/\text{s}\) – foil thermal diffusivity
- \(t_f\) – foil thickness
- \(A_f = 48 \text{ cm}^2\) – utilized area of the foil

IR camera properties:
- \(\sigma_{IR} = 15 \text{ mK}\) – IR camera NET
- \(f_{IR}\) – frame rate of IR camera
- \(N_{IR}\) – number of IR pixels

IRVB properties:
- \(A_{bol}\) – pixel area
- \(f_{bol}\) – frame rate of IRVB

IRVB properties:
- \(A_{bol}\) – pixel area
- \(f_{bol}\) – frame rate of IRVB
- \(N_{bol}\) – # of bolometer pixels

\(S_{IRVB}\) – IRVB noise equivalent power density
- \(\eta_{IRVB}\) – IRVB noise equivalent power

\[
SNR = \frac{S_{signal}}{S_{IRVB}} = \frac{\kappa \cos^4 \theta P_{rad} l_{plasma}}{4\pi k f_{bol} \sigma_{IR} l_{ap-f}^2 V_{plasma}} = \frac{\kappa \cos^4 \theta P_{rad} l_{plasma} A_{ap}}{4\pi k f_{bol} \sigma_{IR} l_{ap-f}^2 V_{plasma}} = \sqrt{\frac{f_{IR} N_{IR} A_{bol}^3}{2 A_f f_{bol}^3}}
\]

Plasma parameters:
- \(L_{plasma}\) – length sight line in plasma
- \(P_{rad}\) – total radiated power
- \(V_{plasma}\) – plasma volume

Pinhole camera properties:
- \(A_{ap} = (1.4)^2 A_{bol}\) – area of aperture
- \(l_{ap-f}\) – distance from foil to aperture
- \(\Theta = 10 - 20\) – angle between sightline and aperture

\(S_{signal}\) – estimated radiated power density on foil

\[
\frac{S_{signal}}{S_{IRVB}} = \frac{P_{signal}}{A_{bol}} = \frac{A_{bol}^4 A_{ap} \cos^4 \theta P_{rad} l_{plasma}}{A_{bol} 4 l_{ap-f}^2 V_{plasma}}
\]

Phantom reconstruction tests (2)

- High reconstruction accuracy near first wall in KSTAR
  - useful for impurity seeding exp. or plasma-divertor detachment

- D-shape + Hot spots in b) X-point, c) inboard and d) outboard divertor

Phantom reconstruction tests (+10% noise)

- High reconstruction accuracy near first wall in KSTAR
  - useful for impurity seeding exp. or plasma-divertor detachment

a) D-shape + Hot spots in b) X-point, c) inboard and d) outboard divertor

Spatial resolution of IRVB tomography

- Spatial resolution of IRVB tomography
  - ~ 9 cm
  - Two gaussian peak with 8 cm gap can be distinguished

Phantom

Reconstruction

Phantom

Reconstruction

Radial profiles

50% of peak
Spatial resolution of IRVB (+10% noise)

✓ Spatial resolution of IRVB tomography

~ 9 cm

: Two gaussian peak with 9 cm gap can be distinguished
(10% noise added to line-integrated signal)