Numerical simulations of GAE stabilization in NSTX-U

E. V. Belova¹, E. D. Fredrickson¹, N. A. Crocker², and the NSTX-U team

¹) Princeton Plasma Physics Laboratory, Princeton NJ, USA
²) University of California, Los Angeles, California 90095, USA
NSTX-U tangential 2nd neutral beam suppresses Global Alfven Eigenmodes (GAEs) [E. Fredrickson, PRL 2017]

- Counter-propagating GAEs are frequently observed in the sub-cyclotron frequency range of $0.1 f_{ci}$ up to $0.5 f_{ci}$ in NSTX and NSTX-U.
- Driven by cyclotron resonance with beam ions.
- New neutral beam sources -> ability to control the fast ion distribution.
- Off-axis neutral beams inject fast ions onto trajectories largely parallel to the magnetic field, with pitch $0.8 < V_{||}/V < 1$.
- Reliable suppression of the counter-propagating GAE when an additional 1.3MW is injected using the outboard beam.
- GAE stabilization has been well documented for many shots.

[Graph showing frequency vs. time with modes stabilized by neutral beams]
Correlation between strong GAE/CAE activity and flattening of the electron temperature profile has been observed in NSTX [Stutman, PRL 2009]

- Intense GAE/CAE activity (0.5-1.1MHz).
- Flattening of $T_e$ profile with
  - increased beam power;
  - beam energy scanned between 60 and 90 keV [Stutman, PRL 2009].
- Was attributed to
  - enhanced electron transport due to orbit stochasticity in the presence of multiple GAEs [Gorelenkov, NF 2010].
  - energy channeling due to CAE coupling to KAW [Belova, PRL 2015].
- Anomalously low $T_e$ potentially can have significant implications for future fusion devices, especially low aspect ratio tokamaks.

Correlation between GAE activity, $T_e$ flattening, and central electron heat diffusivity $\chi_e$ in NSTX H modes with 2, 4, and 6MW neutral beam.
HYM – HYbrid and MHD code

<table>
<thead>
<tr>
<th>Code description</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D nonlinear.</td>
<td>NSTX</td>
</tr>
<tr>
<td>Physical models:</td>
<td>- Sub-cyclotron frequency Alfven eigenmodes (GAE and CAE)</td>
</tr>
<tr>
<td></td>
<td>ICC Theory and Modeling</td>
</tr>
<tr>
<td></td>
<td>- Hybrid simulations of spheromak merging</td>
</tr>
<tr>
<td></td>
<td>- FRC: Effects of beam ions on stability</td>
</tr>
<tr>
<td></td>
<td>- Rotation control</td>
</tr>
<tr>
<td></td>
<td>- n=2 rotational and n=1 wobble modes</td>
</tr>
<tr>
<td></td>
<td>- Resistive MHD &amp; Hall-MHD</td>
</tr>
<tr>
<td></td>
<td>- Hybrid (fluid electrons, particle ions)</td>
</tr>
<tr>
<td></td>
<td>- MHD/particle (one-fluid thermal plasma, + energetic particle ions)</td>
</tr>
<tr>
<td></td>
<td>- Drift-kinetic particle electrons</td>
</tr>
<tr>
<td></td>
<td>Full-orbit kinetic ions.</td>
</tr>
<tr>
<td></td>
<td>For particles: delta-f / full-f numerical scheme.</td>
</tr>
<tr>
<td></td>
<td>Parallel (3D domain decomposition, MPI)</td>
</tr>
</tbody>
</table>
Self-consistent MHD + fast ions coupling scheme

Background plasma - fluid:

\[ \rho \frac{dN}{dt} = -\nabla p + (j - j_b) \times B - n_b (E - \eta j) \]

\[ E = -V \times B + \eta j \]
\[ B = B_0 + \nabla \times A \]
\[ \partial A / \partial t = -E \]
\[ j = \nabla \times B \]
\[ \partial p^{1/\gamma} / \partial t = -\nabla \cdot (\nabla p^{1/\gamma}) \]
\[ \partial \rho / \partial t = -\nabla \cdot (\nabla \rho) \]

Fast ions – delta-F scheme:

\[ \frac{dx}{dt} = v \]
\[ \frac{dv}{dt} = E - \eta j + v \times B \]

\[ w = \frac{\delta F}{F} \quad \text{- particle weight} \]
\[ \frac{dw}{dt} = -(1-w) \frac{d(\ln F_0)}{dt} \]
\[ F_0 = F_0(\varepsilon, \mu, p_{\phi}) \]

\( \rho, V \) and \( p \) are thermal plasma density, velocity and pressure, \( n_b \) and \( j_b \) are beam ion density and current, and \( n_b \ll n_e \) – is assumed.
Self-consistent anisotropic equilibrium including the NBI ions

Grad-Shafranov equation for two-component plasma:
MHD plasma (thermal) and fast ions [Belova et al, Phys. Plasmas 2003].

\[
\frac{\partial^2 \psi}{\partial z^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) = -R^2 p' - HH' - GH' + RJ_{b\phi}
\]

Beam effects

\[
B = \nabla \phi \times \nabla \psi + h \nabla \phi
\]

\[
h(R, z) = H(\psi) + G(R, z)
\]

\[
J_{bp} = \nabla G \times \nabla \phi, G - \text{poloidal stream function}
\]

Modifications of equilibrium due to beam ions:
- more peaked current profile,
- anisotropic pressure,
- increase in Shafranov shift

might have indirect effect on stability.
Fast ions – delta-f scheme: $F_0 = F_0(\varepsilon, \mu, \rho_\phi)$

Equilibrium distribution function $F_0 = F_1(v) F_2(\lambda) F_3(p_\phi, v)$

- $F_1(v) = \frac{1}{v^3 + v_*^3}$, for $v < v_0$
- $F_2(\lambda) = \exp\left(-\frac{(\lambda - \lambda_0)^2}{\Delta \lambda^2}\right)$
- $F_3(p_\phi, v) = \frac{(p_\phi - p_0)^\beta}{(R_0 v - \psi_0 - p_0)^\beta}$, for $p_\phi > p_0$

where $v_0 = 2.5v_A$, $v_* = v_0/2$, $\lambda = \mu B_0/\varepsilon$ – pitch angle parameter, $\lambda_0 = 0.5$-$0.7$, and $\mu = \mu_0 + \mu_1$ includes first-order corrections [Littlejohn’81]:

$$\mu = \frac{(v_\perp - v_a)^2}{2B} - \frac{\mu_0 v_\parallel}{2B}[\hat{b} \cdot \nabla \times \hat{b} - 2(\hat{a} \cdot \nabla \hat{b}) \cdot \hat{c}]$$

$v_d$ is magnetic gradient and curvature drift velocity, $\hat{c} = v_\perp / v_\perp$, $\hat{a} = \hat{b} \times \hat{c}$.

Parameters are chosen to match TRANSP beam profiles.
HYM simulations reproduce frequency range of unstable GAE and CAE modes observed in NSTX

**Experimental analysis:**
Detailed measurements of GAE and CAE amplitudes and mode structure for H-mode plasma in NSTX shot 141398 [N. Crocker, NF 2013].

- **CAEs**: \( f > 600 \text{ kHz} \), and \( |n| \leq 5 \).
- **GAEs**: \( f < 600 \text{ kHz} \), and \( |n| \approx 6-8 \).
- Co- and counter-rotating CAEs with \( f \approx 1.2-1.8 \text{ MHz} \), and \( n=6-14 \) also observed in the same shot [E. Fredrickson, PoP 2013].

**HYM simulations:**
- For \( n=5-7 \) most unstable are counter-rotating **GAEs**, with \( f = 380-550 \text{ kHz} \).
- For \( n=4 \) and \( n=8, 9 \) most unstable are co-rotating **CAEs** with \( f = 870-1200 \text{ kHz} \).

Frequency versus toroidal mode number for unstable GAEs (red) and CAEs (blue), from HYM simulations and experiment, \( f_{ci}=2.5\text{MHz} \).
Simulations have been performed to study the excitation and stabilization of GAEs in the NSTX-U.

- Simulations using the HYM code have been performed for NSTX-U shot #204707 right before (t=0.44s) and shortly after (t=0.47s) the additional off-axis beam injection.
- Plasma and beam profiles have been chosen to match TRANSP profiles for t=0.44s and t=0.47s.
- The beam ion distribution function matches TRANSP data, with pitch distribution in the form $F_b \sim \exp[-(\lambda - \lambda_0(\epsilon))^2/\Delta \lambda(\epsilon)^2]$.

(a) Spectrogram on magnetic fluctuations (n=8-11 counter-GAEs).
(b) Rms magnetic fluctuations;
(c) Injected beam power.

Plasma shape, q- and $n_b/n_0(R)$ profiles for NSTX-U shot 204707 t=0.44 from TRANSP and HYM GS solver + FREE_FIX.
HYM reproduces experimentally observed unstable GAEs

(a) Spectrogram on magnetic fluctuations (n=8-11 counter-GAEs).
(b) Rms magnetic fluctuations;
(c) Injected beam power.

(a) Growth rates and (b) frequencies of unstable counter-GAEs from HYM simulations for t=0.44s. Blue line is Doppler-shift corrected frequencies, points – experimental values.

- HYM overestimates growth rates compared experimental analysis
- Growth rates are sensitive to distribution function parameters – resonance particles are in ‘tail’ of distribution.
NSTX-U simulations: n=10 counter-GAE (t=0.44s)

Radial profile of $\delta B$ and $\delta n$ for n=10 counter-GAE.

Radial profile of $\omega_A/\omega_{ci}$ for n=10, m=0-2; location of GAE.

- Unstable modes are counter-rotating and have shear Alfven polarization.
- Located near min of $\omega_A$. 
Improved $F_{\text{beam}}$ fit reduces growth rates

$$F_{\text{beam}} \sim \exp(-(\lambda - \lambda_0(v))^2 / \Delta \lambda(v)^2)$$

(a) TRANSP fast-ion distribution for $t=0.44$, resonant line for $n=-11$ GAE, estimated $\gamma_{\text{dr}} \sim 0.5%\omega_{ci}$.
(b) HYM fast-ion distribution from $n=-10$ GAE simulations, $\gamma_{\text{dr}} \sim 2.5%\omega_{ci}$. Dots show resonant particles.

(a) Location of resonant particles in phase space: $\lambda = \mu B_0 / \epsilon$ vs $p_\phi$.
(b) Particle weight $w \sim \delta F/F$ vs orbit-averaged parallel velocity. Particle color corresponds to different energies: from $E=0$ (purple) to $E=90\text{keV}$ (red).
Off-axis beam injection strongly suppresses all unstable GAEs

HYM simulations reproduce experimental finding: off-axis neutral beam injection reliably and strongly suppresses unstable GAEs

- The beam ion distribution function with pitch distribution in the form $F_b \sim \exp[-(\lambda - \lambda_0(\varepsilon))^2/\Delta\lambda(\varepsilon)^2]$.
- Additional off-axis beam injection modeled by adding beam ions with distribution $F_{add} \sim \exp[-\lambda^2/\Delta\lambda_a(\varepsilon)^2]$, i.e. with $\lambda_0=0$, $\Delta\lambda_a<\Delta\lambda$ and about 1/3 of the total beam ion inventory.
- HYM shows complete stabilization of $n=7$-$12$ counter-GAEs by additional off-axis beam injection.

Time evolution of magnetic energy of $n=10$ GAE from HYM simulations for $t=0.44s$ (red), and $t=0.47s$ (blue).
Summary and Future Work

- HYM simulations show range of toroidal mode numbers, and frequencies of unstable GAEs that match the experimentally observed GAEs in NSTX-U.
- Growth rate of GAE is sensitive to details of beam ion distribution.
- Simulations reproduce experimental finding, namely it is shown that off-axis neutral beam injection reliably and strongly suppresses all unstable GAEs.
- A robust physical mechanism for stabilizing GAEs - threshold for stabilization for additional beam is less than 25% of total beam power.

Future work:

- Understanding instability conditions for excitation of counter-GAEs – need more general analytical condition.
- Bulk plasma rotation and Hall term can have effect on GAE stability and mode structure.
- Comparison with experimental results including mode structure, saturation amplitudes and etc for several shots.