CHALMERS
UNIVERSITY OF TECHNOLOGY

Alcator C-Mod

Modeling synchrotron radiation images of runaway electrons

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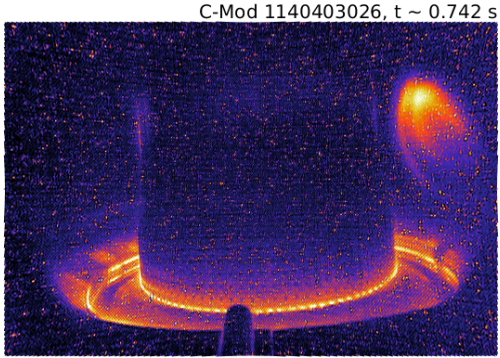
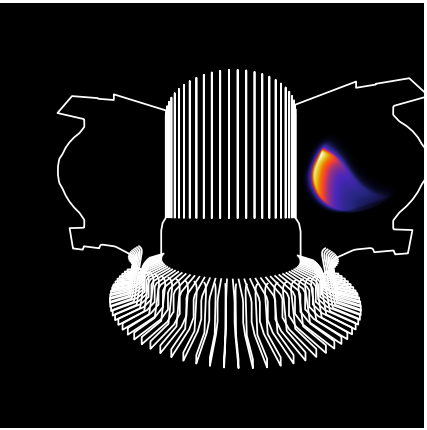
Outline

Q: *Can we measure the runaway electron distribution function?*

1. SOFT
2. Synchrotron radiation
3. Interpretation of synchrotron spot
4. Case studies
    I. DIII-D
    II. Alcator C-Mod



Synchrotron spot in Alcator C-Mod

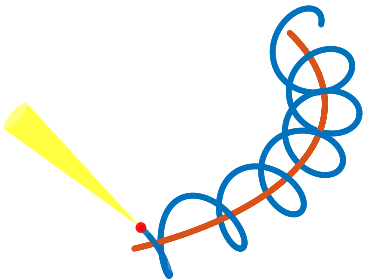


Available on GitHub:
`https://github.com/hoppe93/SOFT`

- **Sparking question:** How do orbits affect the synchrotron radiation?
- **Particle simulations:**
  - Drop particles in outer midplane
  - Follow guding-center orbits
  - Register contributions to a detector
- **Main differences to previous models:**
  - Utilizes numeric magnetic field
  - Considers *detected* radiation (not just *emitted* [1]) and *intensity variations* (not just *shape* [2,3])

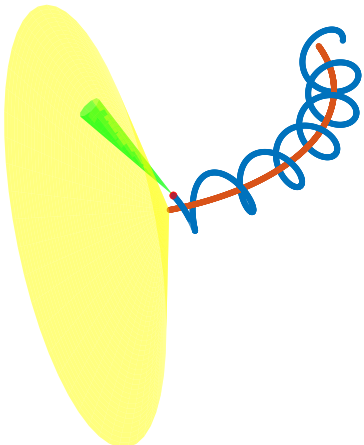
[1] A. Stahl et al., Phys. Plasmas 20 (9), 093302 (2013).
[2] I. M. Pankratov, Plasma Phys. Rep. 22 (6), 535-538 (1996).
[3] R. J. Zhou et al., Phys. Plasmas 21 (6), 063302 (2014).

Synchrotron radiation



- From highly relativistic particles
- Emitted mainly in the direction of motion
- Angular spread around velocity vector $\sim \gamma^{-1}$
- Peaks in visible - IR wavelength ranges

Synchrotron radiation



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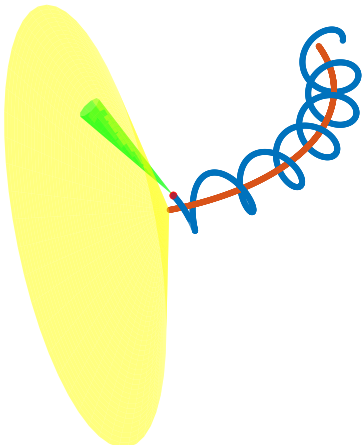
Gyro-average radiation

- Cone of radiation around guiding-center

Synchrotron radiation



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Gyro-average radiation

- Cone of radiation around guiding-center

$\Longrightarrow$ **Only electrons moving towards observer are visible**

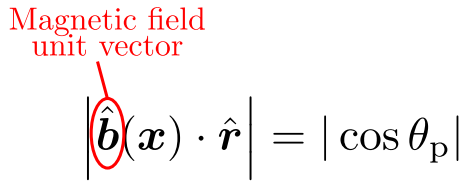
Spot shape equation

$$\left| \hat{\boldsymbol{b}}(\boldsymbol{x}) \cdot \hat{\boldsymbol{r}} \right| = |\cos \theta_{\mathrm{p}}|$$

Reminder:

- Assume: guiding-center follows magnetic field lines
- Cone of radiation around GC; opening angle θ_{p}

Spot shape equation



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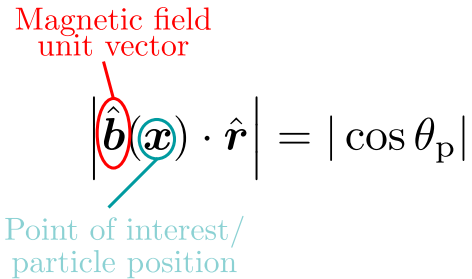

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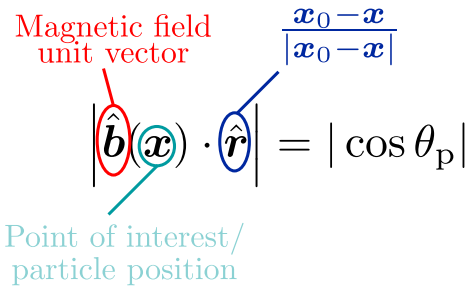

$\boldsymbol{x}_0 =$ Observer location

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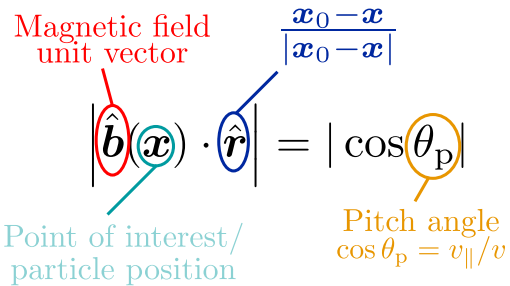

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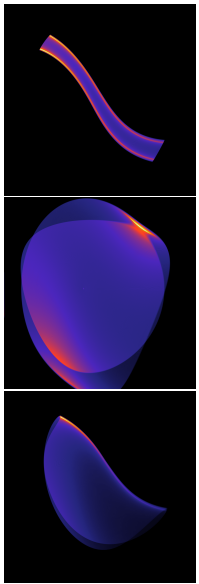
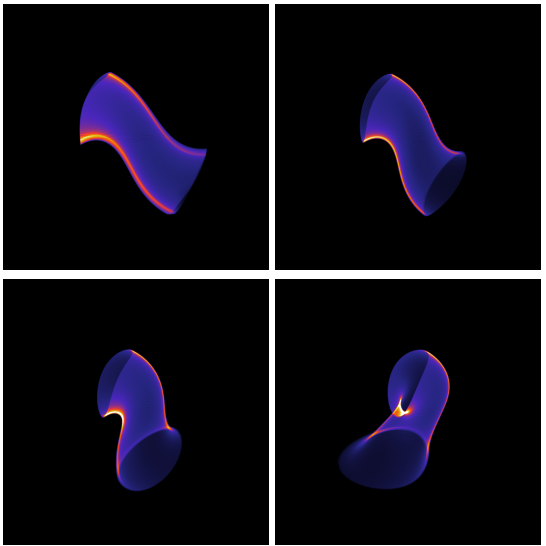
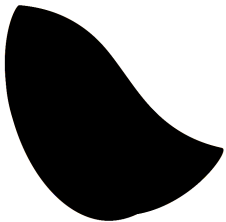
Surface-of-visibility



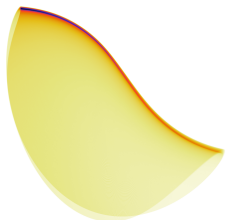


Mechanisms of dominant radiation

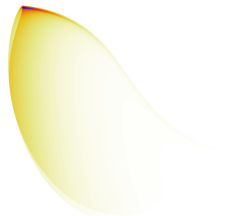
Surface-of-visibility



Line-integration and emission effects



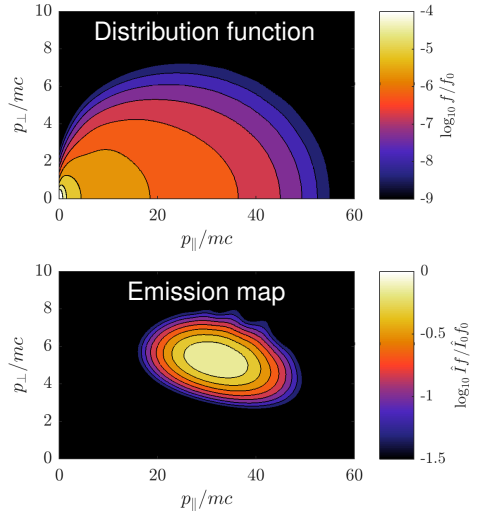
Finite spectral range effects



Mechanisms of dominant radiation

Simple spot shape model could have been okay, except...

- Particles are widely distributed in momentum-space
- Actual spot shape is a superposition of images



Mechanisms of dominant radiation

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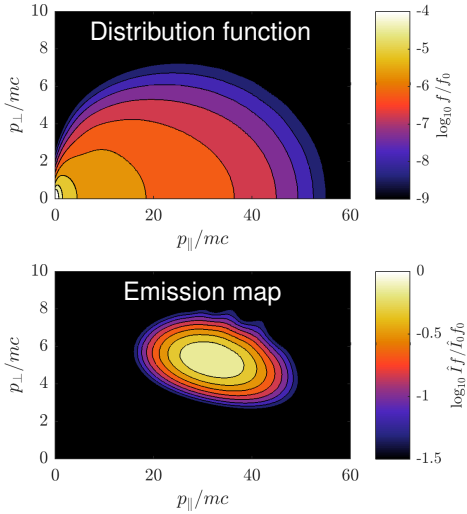
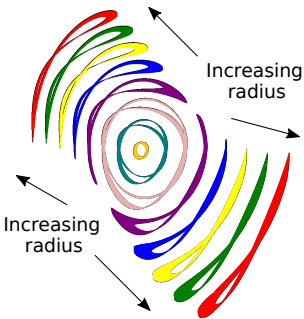
$\Longrightarrow$ Image dominated by line-integration and spectral effects

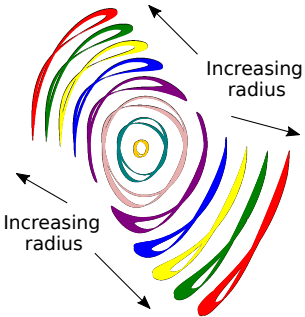


Radius

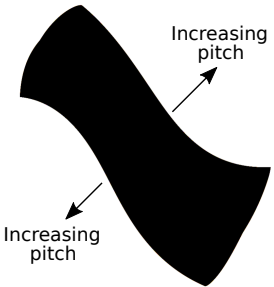


Parameter dependencies

Radius



Pitch angle



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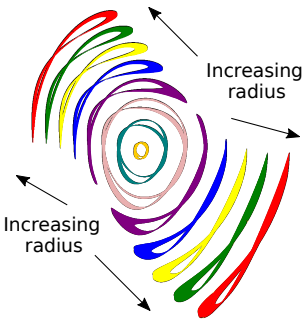

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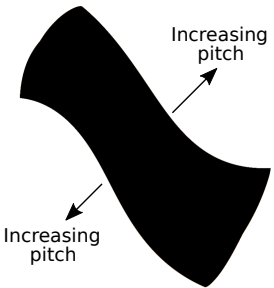
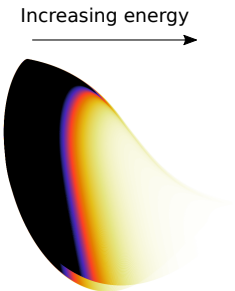


Energy



Parameter dependencies – *Energy dependence*

Assume

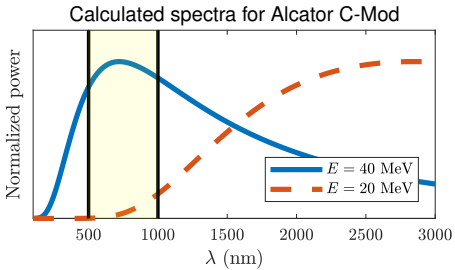
$$B = \frac{B_0 R_0}{R}.$$

In the limit $\lambda \ll \lambda_c$ the emitted power scales with major radius R as

$$P \sim \left(\frac{R}{R_c}\right)^{3/4} \exp\left[-\left(\frac{R}{R_c}\right)^{3/2}\right],$$

$$R_c \equiv \frac{B_0 R_0}{2m}\left(\frac{3e\gamma\lambda\sqrt{\mu}}{\pi c^2}\right)^{2/3}.$$

As $\lambda \to \lambda_c$, the scaling gets weaker.



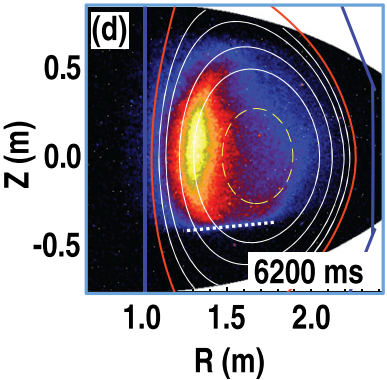
Alcator C-Mod

$$R_c \sim \frac{1}{5} R_0$$

DIII-D

$$R_c \sim \frac{1}{10} R_0$$

Case I: DIII-D



Observations

- Strong radiation from HFS
- Large vertical extent of spot

Experimental image from *C. Paz-Soldan et al., Phys. Plasmas 21 (2), 022514 (2014).*

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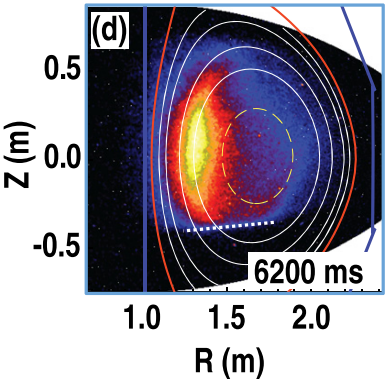

Observations

- Strong radiation from HFS
- Large vertical extent of spot

First conclusions

- Dominant energy between 10-20 MeV
- Dominant pitch angles at least 0.20 rad

Experimental image from *C. Paz-Soldan et al., Phys. Plasmas 21 (2), 022514 (2014).*

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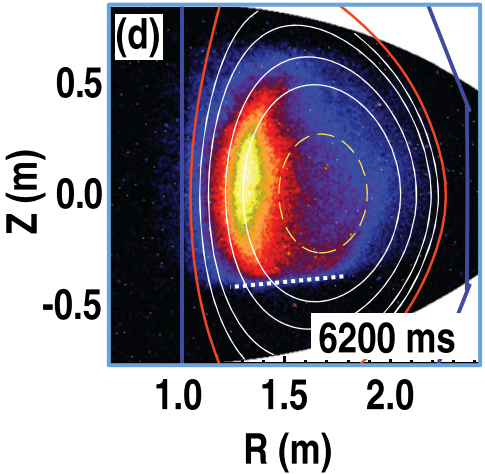

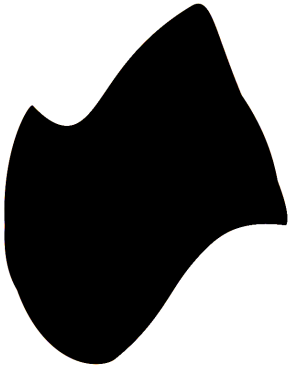

Spot shape

Experimental image from *C. Paz-Soldan et al., Phys. Plasmas 21 (2), 022514 (2014).*

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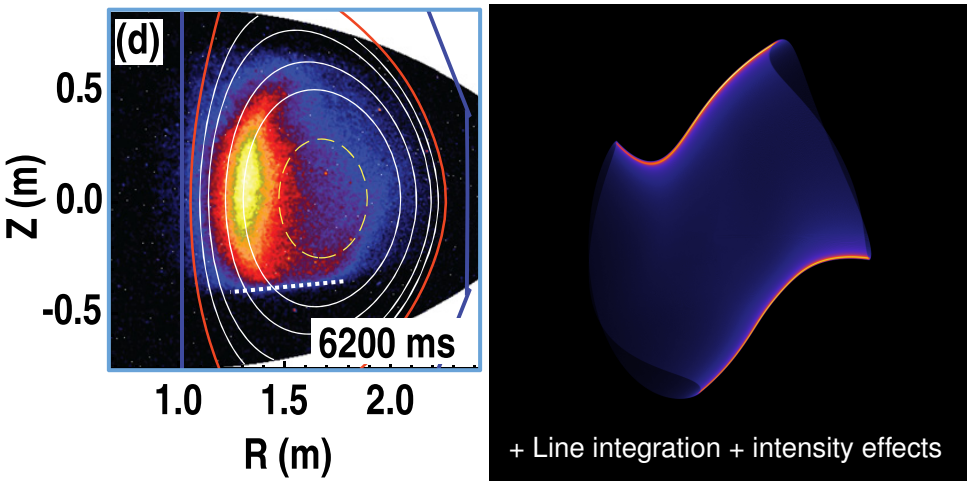

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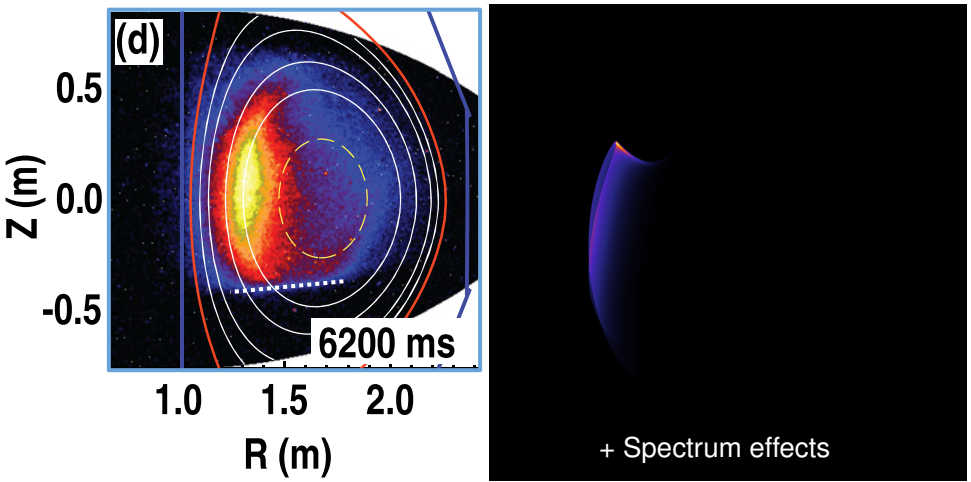

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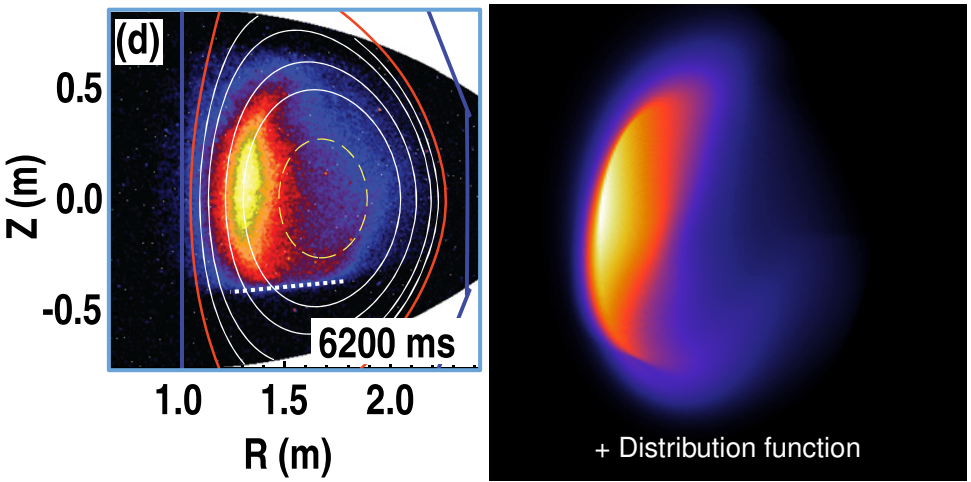

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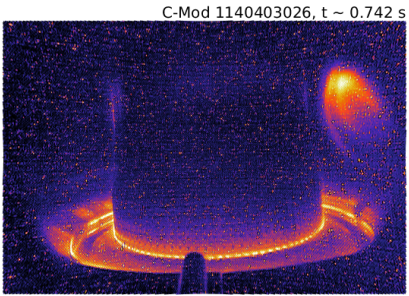
Case II: Alcator C-Mod



Observations

- Horizontally centered radiation
- Three distinct “legs”
- Bright spot at the top

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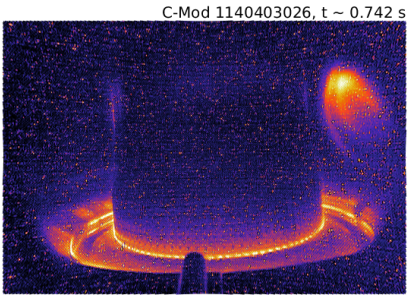

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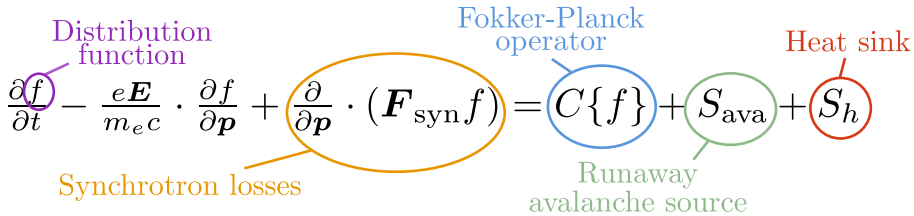
First conclusions

- Weak spot curvature on the left $\Longrightarrow$ dominant pitch angles between 0.10-0.15 rad



- Solves Fokker-Planck equation for relativistic electrons
- 0D in space, 2D in momentum space

[4] M. Landreman et al., Comp. Phys. Comm. 185 (3), 847-855 (2014).
[5] A. Stahl et al., Nucl. Fusion 56 (11), 112009 (2016).

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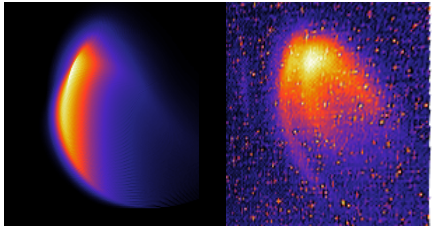
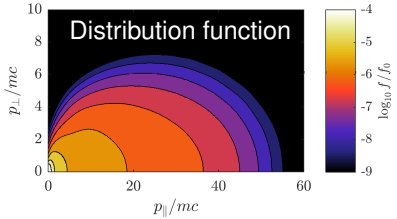
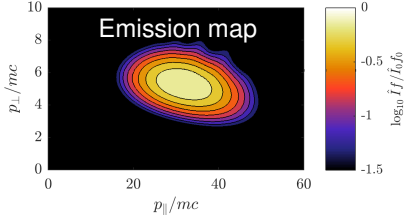






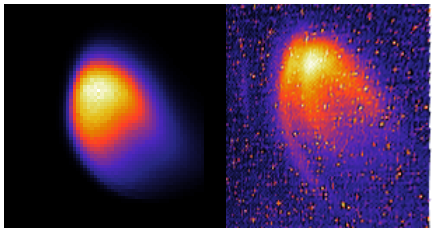
From kinetic simulations:

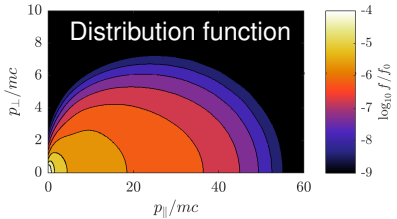
- Dominant energy around 15 MeV
- Dominant pitch angle $\theta_\mathrm{p} \sim 0.15$ rad

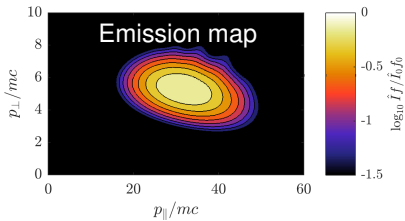
Deduced from image:

- Decreasing radial profile
- Sudden and brief “well” in outer **radial profile**

Case II: Alcator C-Mod







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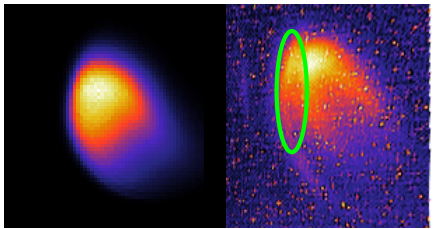

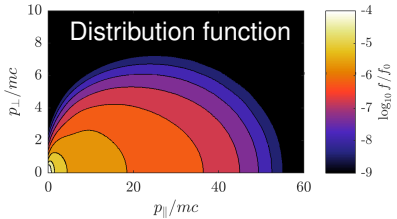

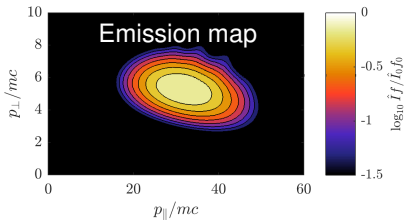

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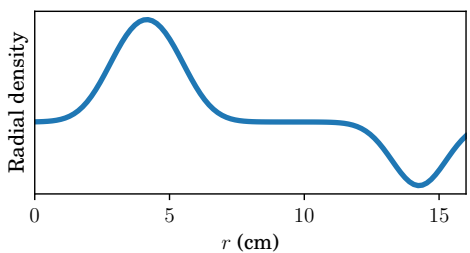
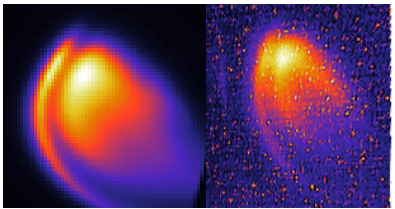
Inversion of the runaway electron distribution function

- Attempt at inverting radial distribution function
- Image comparison + stochastic optimization





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Further reading

[6] M. Hoppe et al., *SOFT: A synthetic synchrotron diagnostic for runaway electrons*, submitted to Nucl. Fusion, arXiv:1709.00674.