Low Frequency Alfven Eigenmodes in Toroidal Plasmas

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Low Frequency Alfven Eigenmodes

• Motivation
  – Loss of energetic particles due to low frequency BAAE & BAE
  – Strong coupling of BAAE & BAE with thermal plasmas: impact on confinement of thermal plasmas, transfer of energy from energetic to thermal ions?

• Existence & excitation of BAAE
  – Discrete BAAE has not been predicted by MHD or kinetic analytic theory
  – Does BAAE exist despite heavy damping by thermal ions?
  – Can BAAE be excited by realistic fast ion density gradient?

• Nonlinear interaction between BAAE & BAE
  – Can BAAE be nonlinearly generated?
  – What is nonlinear dynamics of BAAE?
Both high frequency Alfvén eigenmodes (AE) and low frequency modes (LF)
Up to 45% neutron deficit observed; Half induced by AE; The other half caused by LF modes
LF: Beta-induced Alfven eigenmode (BAE)? beta-induced Alfven-acoustic eigenmode (BAAE)?

G. Kramer & R. Nazikian
Initial Results from GTC Simulation for n=2 LF Modes

- DIII-D LF 40~100kHz
- Unstable BAE-like mode, \( \omega = 61 \) kHz (91kHz in lab frame)
- BAAE-like mode nonlinearly generated \( \omega = 28k\)Hz (58kHz in lab frame)
- LF modes also driven unstable by thermal plasma pressure gradient
Existence of BAAE Verified by GTC

- $T_i \ll T_e$ to minimize BAAE damping
- Frequencies and radial location of three excitation methods agree
- Discrete frequency $\sim$ nearby accumulation point: nonlocal effects
- Direction of mode structure, frequency sweeping in reverse shear, agrees with experiments

Transition from BAE to BAAE for Larger Device $T_i=0.5T_e$

- $a/R=0.29$
- $B=1.91\,T$
- $T_e=2\,keV$  
- $T_i=0.5T_e$  
- $T_f=9T_e$  
- $(n,m)=(4,6)$  
- $\beta_e=2.87\%$

**Diagram:**
- Graph showing $n_f/n_e$ vs. $q$ for $B=1.91\,T$.
- $T_e=2\,keV$, $T_i=0.5T_e$, $T_f=9T_e$, $(n,m)=(4,6)$, $\beta_e=2.87\%$.

**Graph:**
- Comparison of $\omega(\xi/\xi_0)$ for BAE and BAAE modes.
- $\omega(\xi/\xi_0)$ vs. $r/a$ for $m=5$, $m=6$, $m=7$.

**Legend:**
- BAE
- BAAE

**COHERENCE:**
- $\omega_{BAE}$ and $\omega_{BAAE}$ plotted for $R_0(m)$.
- $\gamma_{BAE}$ and $\gamma_{BAAE}$.

**Note:**
- BAE & BAAE coexist.
Polarization of Unstable BAAE and BAE Alfvenic

- All poloidal harmonics of unstable BAAE and BAE are Alfvenic
- Gradually decrease the drive → damped modes.

\[ \frac{E_{\parallel}/E_{\parallel ES}}{(BAAE)} \begin{cases} \ll 1, m=6 \text{ Alfvén} \\ \sim 1, m = 5,7 \text{Acoustic} \end{cases} \]

\[
\frac{E_{\parallel}/E_{\parallel ES}}{(BAE)} \ll 1, m=5,6,7 \text{ Alfvén}
\]

- Perturbative, radially local theory not valid for BAAE
Linear Wave-Particle Energy Exchanges

• BAAE & BAE excited by transferring from fast ion perpendicular energy
• BAAE damped by transferring to thermal ion parallel & perpendicular energy
• BAE damped by transferring to thermal ion perpendicular energy
• BAAE: $\omega_r=0.50 v_i/R_0$, $\gamma=0.04$, $\gamma_{EP}=0.08$, $\gamma_{D\_unstable}=-0.04$, $\gamma_{D\_stable}=-0.22$
• BAE: $\omega_r=2.5 v_i/R_0$, $\gamma=0.28$, $\gamma_{EP}=0.44$, $\gamma_{D\_unstable}=-0.16$, $\gamma_{D\_stable}=-0.05$
• BAAE excited even $\gamma_{EP} \ll \gamma_{D\_stable}$: perturbative theory not valid for BAAE

Nonlinear Generation of BAAE by BAE

- Linearly stable BAAE NL driven by BAE with $n_f = 9.5\% n_e$
- $\gamma_{BAAE} \sim \text{const}$ when BAE saturates
- NL generation of BAAE by BAE in DIII-D?
- Threshold $n_f = 9\% n_e$
- Frequency chirping down
- Amplitude oscillation: saturation not due to profile relaxation

Y. Q. Liu, PhD Thesis
Predictive Capability Requires Integrated Simulation of Nonlinear Interactions of Multiple Kinetic-MHD Processes

- Neoclassical tearing mode (NTM) is the most likely instability leading to disruption, NTM excitation depends on nonlinear interaction of MHD instability, microturbulence, neoclassical transport, and EP effects
Gyrokinetic Toroidal Code (GTC)

• GTC: first-principles, integrated simulation capability for nonlinear interactions of kinetic-MHD processes

• Current capability in a single version:
  ✔ Global 3D toroidal geometry (via EFIT, VMEC, M3D-C1)
  ✔ Microturbulence: 5D gyrokinetic thermal & fast ions, drift-kinetic & hybrid electrons, electromagnetic fluctuations
  ✔ MHD and energetic particle (EP): Alfvén eigenmodes, kink, resistive & collisionless tearing modes
  ✔ Neoclassical transport: Fokker-Planck operators
  ✔ Radio frequency (RF) waves: 6D Vlasov ions
  ✔ Scalable to whole titan computer with GPU, ported to MIC

Phoenix.ps.uci.edu/GTC
[Lin, Science1998]
Multiscale Gyrokinetic Simulation of MHD Modes

- Parallel electric field $E_{||} = -\nabla_{||} \phi - \frac{\partial A_{||}}{\partial t}$

- Need to calculate accurately scalar $\phi$ and vector potentials $A_{||}$

- Ideal MHD $E_{||} \sim 0$. $|E_{||}|/|\nabla_{||} \phi| \sim |E_{||}|/|\frac{\partial A_{||}}{\partial t}| \sim (k_{\perp} \rho_s)^2 \ll 1$

- Physics: cancellation between electrostatic and inductive $E_{||}$

- Canonical momentum used as independent velocity variable to avoid explicit time derivative operation:

  \[
  \left( \nabla_{\perp}^2 - \frac{\omega_{pe}^2}{c^2} - \frac{\omega_{pi}^2}{c^2} \right) \delta A_{||} = \frac{4\pi}{c} (e n_{i0} \delta u_{||ec} - Z_i n_{i0} \delta u_{||ic}),
  \]

  \( (5) \)

- LHS: $|1^{st} \text{ term}|/|2^{nd} \text{ term}| \sim (k_{\perp} d_e)^2 \ll 1$. Small numerical error of RHS leads to large error of $A_{||}$

- Infamous “cancellation problem”: calculate $2^{nd}$-order $\mathcal{O}(k_{\perp}^2 \rho_s d_e)^2$ term from $0^{th}$–order $\mathcal{O}(1)$ equation!
Fluid-Kinetic Hybrid Electron Model

- \( E_\parallel \) from electron parallel force balance. Calculate 2\(^{\text{nd}}\)-order term from 2\(^{\text{nd}}\)–order equation!
- For \( \omega/k_\parallel \ll v_e \), electron response mostly adiabatic (isothermal).
  First, estimate \( E_\parallel \) using massless fluid electron
  \[
  E_\parallel = -\nabla_\parallel (\phi + \phi_{\text{ind}}) = -\frac{T_e}{e n_0} \nabla_\parallel \delta n_e
  \]
  ✓ Vector potential \( A_\parallel \) calculated from \( E_\parallel \)
  ✓ Perturbed flow \( \delta u_e \) from Ampere’s law
  ✓ Perturbed density \( \delta n_e \) from continuity equation
  ✓ Electrostatic potential \( \phi \) from Poisson equation using perturbed density \( \delta n_e \)
- Then, \( E_\parallel \) is corrected by kinetic electron non-adiabatic response using split-weigh scheme to reduce noise
  \[
  \delta f_e = f_M e^{e(\phi + \phi_{\text{ind}})/T_e} + \delta g
  \]
- No tearing mode

A Conservative Scheme Solving Exact DKE

- Calculate exact $E_\parallel$ by separating $A_\parallel$ into adiabatic and non-adiabatic parts: $A_\parallel = A^A_\parallel + A^{NA}_\parallel$
  $$\frac{\partial A^A_\parallel}{\partial t} = \nabla_\parallel \phi_{ind}$$

  $$\frac{e\phi_{ind}}{T_e} = -\frac{e\phi}{T_e} + \frac{\delta n_e}{n_0} - \frac{\delta \psi^A}{n_0} \frac{\partial n_0}{\partial \psi}$$  \hspace{1cm} \text{Alfven wave, IAW, drift wave}

- Define electron adiabatic responses using adiabatic $E_\parallel$
  $$\delta f_A = \frac{e(\phi + \phi_{ind})}{T_e} f_0 + \delta \psi^A \frac{\partial f_0}{\partial \psi}$$

*A conservative scheme of drift kinetic electrons for gyrokinetic simulation of kinetic-MHD processes in toroidal plasmas, J. Bao, D. Liu, Z. Lin, Phys. Plasmas, 2017*

- Mixed-variable gyrokinetics [*Mishchenko et al, PoP (2014)*] separates $A_\parallel$ into “symplectic” and “Hamiltonian” parts, and define ideal MHD as “symplectic” $\phi_{ind} = -\phi$ : Alfven wave
A Conservative Scheme Solving Exact DKE

- Non-adiabatic part of $E_\parallel$ calculated via electron parallel momentum equation (generalized Ohm’s law) using non-adiabatic response
- Recover collisionless tearing mode with current sheet at $d_e$ scale
- Non-tearing modes: current screened by collisionless skin depth $d_e$
- No “cancellation problem”

$$\left( \nabla_\perp^2 - \frac{1}{d_e^2} \right) \frac{\partial A_{\parallel NA}^N}{\partial t} = \frac{1}{d_e^2} \chi_\parallel - c \nabla_\perp^2 (\nabla_\parallel \delta \phi_{ind})$$

\[
\chi_\parallel = -\frac{c}{en_0} b_0 \cdot \nabla \delta P_{\parallel NA} - \frac{c}{en_0 B_0} \delta B_{\parallel NA} \cdot \nabla P_{\parallel 0} - \frac{c}{en_0 B_0} \delta B \cdot \nabla \delta P_{\parallel NA}^N - \frac{c}{B_0} \delta B \cdot \nabla \delta \phi_{ind} \tag{III}
\]

\[
+ \frac{c}{B_0} \delta B \cdot \nabla \langle \phi \rangle - \frac{cm_e}{en_0} \nabla \cdot \left[ n_0 \delta u_{\parallel le} \left( 3 V_c + V_g \right) + n_0 u_{\parallel 0} V_E \right] - \frac{cm_e}{en_0} \nabla \cdot \left( n_0 \delta u_{\parallel le} V_E \right) \tag{VII}
\]

\[
+ \frac{c}{en_0} \frac{P_{\parallel 0} - P_{\parallel 0}}{B_0^2} \delta B \cdot \nabla B_0 + \frac{c}{en_0} \frac{\delta P_{\parallel NA}^N - \delta P_{\parallel NA}}{B_0^2} B_0 \cdot \nabla B_0 \tag{IX}
\]
Compressible Magnetic Perturbations via Perpendicular Force Balance for Slow Modes

\[
\frac{\delta B_{\parallel} B_0}{8\pi} \left\{ 1 + \beta_e + \beta_i \left[ I_0 (k_\perp \rho_i^2) - I_1 (k_\perp \rho_i^2) \right] \exp (-k_\perp \rho_i^2) \right\}
\]

\[
\frac{\beta_i B_0^2 e \delta \phi}{16\pi T_i} \left\{ \left[ I_0 (k_\perp \rho_i^2) - I_1 (k_\perp \rho_i^2) \right] \exp (-k_\perp \rho_i^2) - 1 \right\}
\]

\[
= -\pi \Omega_e^2 \int d\mu dv_{\parallel} B_0 \int_0^{\rho_e} F_{e,gyro}^r r dr - \pi \Omega_i^2
\]

\[
\times \int d\mu dv_{\parallel} B_0 \left\langle \int_0^{\rho_i} F_{i,gyro}^r r dr \right\rangle.
\]

When \( k_\perp \rho_i \ll 1 \), Eq. (13) reduces to

\[
\frac{\delta B_{\parallel} B_0 (1 + \beta_e + \beta_i)}{4\pi} = -\delta P_{e,\perp} - \delta P_{i,\perp},
\]


- Effects of \( \delta B_{\parallel} \) on TAE linear dispersion very small
Zonal Fields Generation and Nonlinear MHD

- ZF in TAE & KBM generated via 3-wave coupling, in contrast to modulational instability in ITG
- Explosive growth of localized current sheet in TAE & KBM by NL ponderomotive force: MHD forward cascade?
Conclusions and Plan

• BAAE can be excited by realistic fast ion density gradient despite strong damping by thermal ions
  – Non-perturbative and nonlocal theory needed for BAAE

• BAAE can be nonlinearly generated by BAE
  – Beyond linear eigenmode paradigm for AE physics and EP transport

• Plan: Integrated Simulation of Energetic Particles in Burning Plasmas
  – SciDAC ISEP (UCI, GA, PPPL, ORNL, LBNL, LLNL, PU, UCSD, UT), 2017
  – First-principles simulations (GTC, GYRO, M3D-K, TAEFL) of AE turbulence and EP transport with multi-physics (coupling with microturbulence & tearing modes) and at long time scale (intermittency & transient)
  – Reduced EP transport models (CGM, RBQ)
  – Verification & validation
  – EP module for whole device modeling: ISEP framework
  – Computational partnership: workflow, solvers, optimization, portability
  – (2 postdocs @ UCI)
AE Saturation & EP Transport Benchmark

- 8 GK & MHD codes
- 3 reduce EP transport models
- Linear benchmark nearly done
- ~ 6 months: AE saturation benchmark
- ~ 1 year: EP transport benchmark