ORBIT modelling of fast particle redistribution induced by sawtooth instability

D. Kim, M. Podestà, D. Liu\(^1\) and F. M. Poli

Princeton Plasma Physics Laboratory

\(^1\)University of California, Irvine

15th IAEA Technical Meeting on Energetic Particles in Magnetic Confinement Systems
Princeton, 5-8 Sep 2017
Conventional sawtooth models are not sufficient for transport modelling

• Sawtooth instability
  – Periodic fast relaxations of plasma parameters (e.g. $T_e$) followed by a slower recovery in the central region ($q < 1$)
  – Magnetic reconnection can be described using full [Kadomtsev] or partial [Porcelli] reconnection model

• TRANSP [Hawryluk] modelling using reconnection models
  – Free parameters (e.g. partial reconnection fraction) in the models need to find an optimum set for each case
  – Parameter setting cannot be self-consistently determined
  – Cannot reproduce experimental results with a fixed parameter setting
    ➔ experimental results are presented in D. Liu’s poster (P-16)
Sawtooth models presently in TRANSP appear inadequate to recover experimental results. Both assumptions result in nearly same drop in simulated neutron rate!
TRANSP results indicate significant effect of thermal profile evolution on neutron rate

- TRANSP run using thermal profiles from conditional average reconstruction
  - Sawtooth model ON for q-profile evolution
  - Sawtooth model OFF for fast ions

- Little/no modulation of fast ion profile
- Significant drops are observed in simulated neutron rate
  - Relative drop from simulation comparable with measured values
  - Motivates improved analysis strategy to unfold thermal vs fast ion effects
Can effect of fast particle transport improve sawtooth model?

• Introduction of kick model [Podestà]
  – Kick model provides more flexibility to adjust parameters in TRANSP for each sawtooth cycle
  – ORBIT [White 84] estimates evolution of \((E, P_\zeta, \mu)\) of each particle to compute the transport probabilities \(p(\Delta E, \Delta P_\zeta | E, P_\zeta, \mu, A)\)

• Improvement of sawtooth model?
  – ORBIT simulations can be a guidance to develop a more comprehensive model for fast particle transport by sawteeth
  – Improvement of sawtooth modelling is anticipated by implementation of a fast particle model into TRANSP as self-consistent modelling including more physics would be possible
  – NSTX-U data can guide model development and be used for validation
Simulation setting

• Simulation code: ORBIT
  – Hamiltonian guiding center particle motion code
  – Use of numerical equilibrium, field perturbations in flux coordinates (Boozer coordinate in this work)
  – Analyzing test particle transport (especially energetic ions)

• Target discharge: NSTX-U #204083
  – Equilibrium data (eqdsk file from TRANSP) from one time slice before sawtooth crash (1093ms)
  – Initial distribution of fast particles from TRANSP (NUBEAM)
  – Number of particle for calculation: 10,000
  – Radial displacement for the application of linear perturbation from sawtooth instability (constant in time)
Displacement from sawtooth crash induces perturbed magnetic field $\delta B$

- Perturbation magnetic field from displacement
  \[
  \delta \vec{B} = \nabla \times (\tilde{\xi} \times \vec{B})
  \]

- Radial displacement model from [Farengo]
  - $\xi(\rho, t, \theta, \zeta) = \sum \xi_{m,n}(\rho, t) \cos(n\zeta - m\theta - \omega t)$
  - $(m, n)$: poloidal and toroidal mode numbers
  - $(\theta, \zeta)$: poloidal and toroidal angles
  - $(1,1)$ mode is applied and temporal change of displacement is not considered

Figure from [Farengo]
In ORBIT code, perturbation is defined using $\alpha$, not $\xi$.

- Perturbation in ORBIT: \[ \delta\vec{B} = \nabla \times (\alpha \vec{B}) \]
  - $\xi$ can be transformed into $\alpha$ [White 13]
  - Radial component of resultant perturbation is equivalent to $\xi$ model

\[ \delta\vec{B} \cdot \nabla \psi_p = \sum_{m,n} \frac{mg + nI}{J} \alpha_{m,n} \cos(n\zeta - m\theta - \omega t) \]

- Mode amplitude is prescribed for $\xi$ and is used for $\alpha$ after normalization.
Given (1,1) mode perturbation brings (2,2) mode like island structure

• Need to modify perturbation form?
  – Shape: cut-off outside $q=1$ surface, modification of central part
  – Central value
  – Modified range of central region
Modification of perturbation profile

Centre shape

Central value

Shape change position

1) $\xi_0 = 0, 0.2\rho_{q=1}$

2) $\xi_0 = 0$

$\xi_0 = 0.25$

$\xi_0 = 0.5$

3) $0.1\rho_{q=1}$

$0.2\rho_{q=1}$

$0.5\rho_{q=1}$

Shape, $0.2\rho_{q=1}$

Shape, $\xi_0 = 0$
Modification of perturbation profile
- Central region shape test

• $\xi$ decreases inside $0.2 \rho_{q=1}$, $\xi_0 = 0$
• Island is formed where $\alpha$ has large gradient
• Case 3 (linear change) brings perturbation at the center but not enough to form an island
Modified perturbation shape

• The resultant $\alpha$ shape is similar to that from [Zhao]
• Set as the default perturbation shape for the further analysis with different mode amplitude (default: 0.01)
Particle energy is not changed significantly by sawtooth

- The final energy is not deviated much from the initial energy
  - With 100 times different amplitude, the change is not substantial
  - Relative change can show some variation among low energy particles

![Graph showing the relationship between initial and final energy](image)

- For different amplitudes (amp $\zeta$) of sawtooth, the change in final energy $E_f$ remains close to the initial energy $E_i$.

$$E_f \approx E_i$$

Initial energy $E_i$ is varied from 10 to 100 keV, and the change in energy is shown for different amplitudes.
Particles are redistributed depending on the mode amplitude

- As perturbation amplitude increases, particle redistribution becomes more significant
- Smaller amplitude case, initially low energy particles are more redistributed while particles in all energy level are affected by larger amplitude sawtooth
Redistribution of different types and energy level particles

- In smaller amplitude case (0.01), particle redistribution is weak

  - Lower energy level [10 ~ 30keV] particles have more redistribution than higher energy [50 ~ 70keV] case
Redistribution of different types and energy level particles

- With larger amplitude (0.1), particles with low and high energy are redistributed

  For both low and high energy level, particles are strongly affected by sawtooth and redistributed
Relative change of neutron rate – comparison with experimental result

• Estimate relative neutron rate change
  – Using deuterium density at each fast particle position, cross section and particle energy of each particle energy

\[
\Delta = \frac{\sum_{k} n_{df,k} S_{f,k} \sqrt{E_{f,k}} - \sum_{k} n_{d0,k} S_{0,k} \sqrt{E_{0,k}}}{\sum_{k} n_{d0,k} S_{0,k} \sqrt{E_{0,k}}} \quad (k = \text{fast particle index})
\]

![Graph 1](image1)

![Graph 2](image2)
Relative change of neutron rate – comparison with experimental result

- Relative neutron rate change from experiment
  - Difference between neutron rate post and pre crash is normalized by neutron rate before sawtooth crash
  - Depending on calibration parameter, values are slightly different

- Perturbation amplitude scan
  - $\Delta$ calculation using $\xi$ amplitude of [0.001 to 0.1]
  - In lower amplitude case, $\Delta$ is almost constant since fast particles are not redistributed much
  - Amplitude of between [0.045 0.1] can reproduce experimental values
Relative change of neutron rate – comparison with experimental result

- Radial potential amplitude scan
  - $\Delta$ calculation using potential amplitude of [0 to 10] keV
  - With finite potential amplitude, a simple exponential radial potential profile is added
  - With fixed perturbation amplitude of 0.01, $\Delta$ does not vary with potential
  - Mode frequency is fixed to 10 kHz
Test of two mode case: (1,1)+(2,2) modes
- perturbation profile

• (2,2) mode based on [Farengo] is added to (1,1) mode
  – Mode amplitude is set to 1/3 of (1,1) mode and twice mode frequency
  – Modification of profile is not applied to (2,2) mode perturbation profile
    ➔ additional island structure is found from Poincaré plot
Test of two mode case: $(1,1)+(2,2)$ modes - relative neutron rate change

- Perturbation amplitude/potential scan
  - Two modes case has the same trend as $(1,1)$ mode case
  - Since the mode amplitude of $(2,2)$ mode is only $1/3$ of $(1,1)$ mode amplitude, the effect seems small
Summary

- Linear perturbation $\alpha$ is implemented into ORBIT code
  - Transformation from $\xi$ is applied
  - Perturbation shape is modified to induce (1, 1) mode island at q=1 surface
- Energy and particle redistribution due to sawtooth crash is investigated
  - Regardless the amplitude of perturbation, $E$ is not changed significantly while $P_\xi$ variation becomes significant with increase of mode amplitude
  - For low amplitude, low initial energy particles are redistributed more while all energy level are affected by high amplitude perturbation
- Relative change of neutron rate during sawtooth crash is compared with experimental value
  - $\Delta$ value increases as mode amplitude grows from amplitude of 0.02 (below $\Delta$ stays almost similar level)
  - Experimental value can be reproduced by amplitude range of [0.045 0.1]
  - Modification of potential does not have significant effect on $\Delta$ value
  - (2, 2) mode is added to (1, 1) mode but amplitude and potential scan results are almost the same as (1, 1) only mode case
References

[Farengo] R. Farengo et al., “Redistribution of high energy alpha particles due to sawteeth with partial reconnection”, NF 2013 (043012)


