A New Understanding of the Bootstrap Current in Steep Edge Pedestal

R. Hager\textsuperscript{1} and C. S. Chang\textsuperscript{1}

\textsuperscript{1}Princeton Plasma Physics Laboratory, Princeton, NJ, USA

Corresponding Author: rhager@pppl.gov

Abstract:

Contrary to conventional neoclassical theory, global XGCa simulations of realistic steep pedestal plasmas reveal an enhanced role of trapped electrons in the plateau-collisional regime even in the edge pedestal of conventional aspect ratio tokamaks. When the ion orbit width becomes comparable to the pedestal width, the bootstrap current is generally found to be lower than that from analytical formulas local neoclassical simulations. A new XGCa-based bootstrap current formula for the steep pedestal region is proposed and its implications on pedestal stability are discussed.

1 Introduction

The bootstrap current is an important actor in the confinement and stability of tokamak fusion plasmas, especially in the steep edge pedestal plasma [1, 2]. Since it is difficult to measure the bootstrap current profile in experiments, many applications such as equilibrium reconstruction, MHD stability analysis, and plasma control rely on analytic formulas to model the bootstrap current. The formula developed by Sauter et al. [3] is one of the most commonly used such formulas. But most of the previous theoretical and numerical studies used assumptions that are valid only in the core plasma, e.g. the small orbit width approximation $\rho_b \ll L_\nabla$ (also called “local regime”) with $\rho_b$ being the ion orbit width and $L_\nabla$ the plasma gradient scale length. In the steep edge plasma, $\rho_b$ can be comparable to the $L_\nabla$ (“non-local regime”), the trapped particle fraction becomes significant, and the plasma distribution functions can deviate significantly from a Maxwellian, so that the assumptions made in earlier studies break down. A few studies that relaxed some of the restrictions of earlier work exist. Kagan and Catto [4] discusses finite orbit width corrections to the ion current arising from the strong radial electric field $E_r$ in the pedestal plasma but is limited to large aspect ratio. Landreman and Ernst [5] investigated the bootstrap current numerically allowing for $\rho_b$ to be comparable to the density gradient length $L_n$ but still required $\rho_b$ to be much smaller than the ion temperature gradient length $L_T$. Koh et al [6] used a previous version of the XGC0 global drift-kinetic code to construct an improved bootstrap current formula for the steep pedestal region. But that version of XGC0 turned out to suffer from a small inaccuracy in its linearized collision
operator that caused an increase of the bootstrap current. Therefore, we followed up on Koh et al.’s work using the global, gyrokinetic-neoclassical code XGCa and developed a new formula for the bootstrap current [2].

2 The XGCa code

XGCa is a 5D total-δf [8], gyrokinetic, particle-in-cell (PIC) code in toroidal geometry with axisymmetric field solver for the simulation of neoclassical transport in tokamak devices. XGCa does not employ a local approximation and retains full finite orbit-width effects. The unique feature of the XGCa codes is the ability to include the whole volume inside the reactor wall in the simulation, i.e. from the magnetic axis and across the separatrix and into the scrape-off layer (SOL). The neoclassical radial and poloidal electric fields are evaluated self-consistently by solving the gyrokinetic Poisson equation in two dimensions (R and Z) on an unstructured triangular mesh. Collisional effects are evaluated using a fully nonlinear, multi-species Fokker-Planck-Landau collision [9].

In order to have a representative sample of plasma equilibria for fitting our improved bootstrap current formula, we used various realistic diverted magnetic geometries representing various tokamaks and included some circular, fixed-boundary equilibria. Together, these cases span a wide range of edge aspect ratios (1.4-4), triangularities (0-0.39), and elongations (1-2.2).

Apart from XGCa, we also used the neoclassical solver NEO [11, 12] inside the magnetic separatrix for a cross-verification of our XGCa results. NEO is a local code that solves an expansion of the drift-kinetic equation for small ion orbit width $\rho_b/L_T \ll 1$. An example of our cross-verification tests of XGCa and NEO in NEO’s validity regime is shown in Fig. 1. Figure 1 a) shows the density and temperature profiles of this simulation, and Fig. 1 b) shows the corresponding bootstrap current. The magnetic equilibrium used for this case is a circular fixed-boundary Grad-Shafranov solution generated with the code.

FIG. 1: Bootstrap current in the local regime in a circular, fixed-boundary Grad-Shafranov equilibrium. a) Density and temperature vs. normalized poloidal flux $\psi_N$. b) Bootstrap current density calculated with XGCa, NEO, the Sauter formula, and the improved formula, Eq. (2). [Reproduced from R. Hager and C.S. Chang, Phys. Plasmas 23, 042503 (2016) with the permission of AIP Publishing.]
FLOW \cite{12}. This case is in the local regime characterized by a small ratio of \(\rho_b/L_\nabla \leq 0.2\), where \(L_\nabla\) can be the density gradient scale length \(L_n\) or the ion temperature gradient scale length \(L_{T_i}\). The choice of this threshold is justified in Sec. 3. In this regime, the local approximation used by NEO is valid, and the bootstrap currents calculated with XGCa and NEO agree very well. The reason for the difference between the numerical results and the Sauter formula is not surprising because the electron collisionality \(\nu_{ce}\) is well above one, where the Sauter formula is known to suffer from a loss of accuracy \cite{13}.

3 New physics of the bootstrap current in the H-mode pedestal

The physics of the bootstrap current in the steep edge pedestal differs from the core plasma in two important ways. The first difference is the large trapped particle fraction \(f_t\) in the edge region of realistic tokamak plasmas. Even in conventional aspect ratio tokamaks such as DIII-D, C-Mod, and ASDEX-Upgrade, the trapped particle fraction in the H-mode pedestal is around 70\%, whereas \(f_t \sim \sqrt{2}\epsilon \ll 1\) in the low inverse aspect ratio core plasma. The second big difference is that the gradient scale length \(L_\nabla\) (effectively the pedestal width) and the ion orbit width can be comparable so that the local approximation formally breaks down.

We discuss the significance of the high trapped particle fraction first, anticipating that the ion contribution to the bootstrap current in the laboratory frame of reference remains small throughout the H-mode pedestal (Fig. 2). A well known, 40 year-old result of local neoclassical theory at high aspect ratio is that the contribution from the trapped particles to the bootstrap current is a factor of \(\epsilon\) smaller than the passing particle current \cite{14}, i.e. negligible in the core plasma. The small trapped particle current in the core originates is due to good cancellation of the contributions from co- and counter-moving (with respect to the magnetic field) particles. To the contrary, XGCa simulations show that the trapped particle current can become significant in the H-mode pedestal of modern tokamaks, where \(\epsilon \geq 0.3\) due to incomplete cancellation between the co- and counter-traveling electron orbits and the largeness of the trapped particle fraction. Examples are shown in Figs. 3 a) for circular case discussed above (Fig. 1) and b) for NSTX discharge 132543. In both cases, the majority of the bootstrap current around its maximum location (at \(\epsilon = 0.51\) in the circular equilibrium and at \(\epsilon = 0.68\) in NSTX) is carried by trapped electrons. Even in C-Mod, the trapped particle current can be responsible for up to one third of the total bootstrap current \cite{17}. An interesting feature of Fig. 3 a) is that the passing electron
current even becomes negative for $\psi_N \geq 0.62$. In the trapped region, $\tilde{j}_b(\psi, \lambda)$ decays only slowly starting from the trapped-passing boundary due to incomplete cancellation of the co- and counter traveling orbits. This and the size of the trapped region account for the large trapped electron current. Negative current is clearly confined to the passing region. It is caused by friction between the passing electron flow and the enhanced ion flow visible in Fig. 2 at $\psi_N > 0.62$. In order to contribute the same-directional current, electrons and ions flow in the opposite directions. With the smallness of the passing region and the $\nu_{e^*} > 1$, the passing electrons become much more susceptible to collisions because they are scattered from the passing into the trapped region and back more frequently.

Although it does not appear explicitly in common bootstrap current formulas, the radial electric field is a crucial part in the physics of the bootstrap current, especially in the H-mode pedestal. The reason for its importance is the simple fact that it is a radial force just as the radial pressure gradient, and thus a source of the bootstrap current in a non-thermal plasma in the steep gradient region. This is evident in the local neoclassical equation for the parallel ion flow [14],

$$nu_{\parallel i} = -\frac{I}{m\Omega_i} \left( \frac{d\langle p_i \rangle}{d\psi} + en\frac{d\langle \phi \rangle}{d\psi} \right) + K(\psi)B,$$

where $\Omega_i$ is the ion cyclotron frequency, $B$ is the magnetic field strength, $I = RB_T$ with the toroidal field $B_T$, $n$ is the density, $\phi$ is the electrostatic potential, and $K(\psi)$ is a flux function determined by kinetic physics. A similar equation exists for the electrons with the electric field term having opposite sign. In the local regime, the forces from the ion pressure gradient and $E_r$ tend to cancel. This means that the bootstrap current drive of the ions due to the ion pressure gradient is effectively transferred to the electrons via $E_r$, and the remaining ion flow is small. In the total current, the sum of ion and electron flows, the $E_r$ term cancels. This effect can be seen in Fig. 2.
In the steep edge pedestal, however, the cancellation of the $E_r$ term is incomplete when the ion orbit width becomes comparable to the pedestal width because the orbit averaged effect of the radial electric field on ions becomes different from the effect on electrons. Through this mechanism, non-local effects can influence the bootstrap current regardless of the small ion contribution. Neoclassical codes using the local approximation cannot reproduce this finite orbit-width effect correctly even when the radial electric field from a non-local simulation is supplied as input. Figure 4 shows the ion and electron contributions to the bootstrap current in C-Mod discharge 1120803014, with the normalized poloidal flux on the lower and $\rho_b/L_{T_i}$ on the upper horizontal axis. Inside the H-mode pedestal, where $\rho_b/L_{T_i} \leq 0.2$, the ion and electron currents obtained with XGCa and NEO show good agreement. But in the steep pedestal region ($\rho_b/L_{T_i} > 0.2$), XGCa and NEO results start to deviate. The large ion current in the laboratory frame in the NEO simulation contradicts the local approximation that requires the ion current to be small, showing that a local theory or simulation cannot calculate the plasma flow accurately.

Other sources of finite orbit-width effects are not discussed in detail here are the radial variation of the ion collision frequency, safety factor, and parallel ion and electron flows, as well as the non-Maxwellian nature of the ion distribution function in the steep pedestal.

4 An improved bootstrap current formula

Based on numerous XGCa simulations of a variety of different plasma equilibria, we constructed a bootstrap current formula specifically for the steep edge pedestal that is based on the widely used formula by Sauter et al. [3],

$$j_b = -(\beta_{col} + \beta_{\nabla T_i}) \frac{I_{pe}}{B_0} \left\{ \gamma_{31} L_{31} \frac{1}{p_e} \frac{dP}{d\psi} + \frac{\gamma_{32} L_{32}}{T_e} \frac{dT_e}{d\psi} \right\} + (\gamma_{44} L_{44})(\gamma_{55} \alpha_i) \frac{1}{Z T_e} \left\{ 1 - 2\Delta \psi \left( -\frac{3}{2} L_{T_i}^{-1} + L_n^{-1} + L_q^{-1} \right) \right\} \frac{dT_i}{d\psi},$$

In this formula, $\psi$ is the poloidal magnetic flux, $I(\psi) = RB_0$, $B_0$ is the magnetic field on the magnetic axis, $T_{i/e}$ are the ion and electron temperatures, $p_e$ is the electron pressure, $P$ is the total pressure. $L_{T_i}$, $L_n$, and $L_q$ are the gradient scale lengths of the ion temperature, the density and the safety factor, where $L_X^{-1} \equiv X^{-1}dX/d\psi$, and $\Delta \psi$ is the ion orbit width $\rho_b$ in units of poloidal flux. We only explain the meaning of the different coefficients in Eq. (2) here. For the exact definitions we refer to Ref. [7]. The coefficients $L_{31}$,
FIG. 5: Accuracy of the new formula for a diverse data set of steep pedestal plasmas [XGCa results vs. Eq. (2)]. [Reproduced from R. Hager and C.S. Chang, Phys. Plasmas 23, 042503 (2016) with the permission of AIP Publishing.]
5 Implications for pedestal stability

Since the bootstrap current in the steep edge pedestal is so large that it can be a significant part of the total plasma current, it is beneficial for the plasma because it contributes to the poloidal magnetic field that confines the plasma, but it can also destabilize current driven instabilities and other modes whose growth rate depends on the magnetic shear. In case of the C-Mod equilibrium discussed above, the total plasma current is significantly lower (by 12.7% at $\psi_N = 0.96$ and by 19.3% at $\psi_N = 0.97$) when the improved formula Eq. (2) is used (Fig. 6). Lower plasma current implies higher magnetic shear, which according to Ref. [2] destabilizes intermediate $n$ kinetic peeling balloning modes.

6 Summary

We studied the bootstrap current in the steep edge pedestal using the global code XGCa allowing for $b_L n$ and $b_L T_i$. Our study shows that, contrary to conventional neoclassical theory, the trapped electron contribution to the bootstrap current can be significant in tokamak edge pedestal. We note here that the non-cancellation of the bootstrap current between co- and counter-flowing trapped electrons can be noticed in Fig. 11 of Ref. [16] at small aspect ratio. In the non-local regime, the bootstrap current calculated with XGCa is generally lower (by up to 15%) than the one calculated with the NEO code and much lower than Sauter formula. Our improved bootstrap current formula that is based on XGCa simulations of numerous magnetic geometries and plasma conditions reproduces XGCa results much better than the Sauter formula while being equally simple to use. We discussed the effect of the our modified bootstrap current formula on the total plasma current profile to assess the implications for pedestal stability. In an ongoing effort, we are using a fluid-electron kinetic-ion hybrid version of the XGC1 code to analyze how the pedestal stability changes with different bootstrap current profiles.

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