ANOMALOUS ABSORPTION AND EMISSION IN ECRH EXPERIMENTS DUE TO PARAMETRIC EXCITATION OF LOCALIZED UH WAVES

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Abstract

We analyze saturation of the low-threshold absolute parametric decay instability of an extraordinary pump wave leading to the excitation of two upper hybrid (UH) waves, only one of which is trapped in the vicinity of a local maximum of the plasma density profile. The secondary decay of the localized daughter UH wave are treated as the moderators of the primary two-plasmon decay instability. The reduced equations describing the nonlinear saturation phenomena are derived.

The general analytical consideration is accompanied by the numerical analysis performed under the experimental conditions typical of the off-axis X2-mode ECRH experiments at TEXTOR. The possibility of substantial (up to 10%) anomalous absorption of the pump wave is predicted. The experimental consequences of the decay are discussed. In particular, the level of the pump sub-harmonics emission at 1/2 and 3/2 the pump frequency associated with the predicted instability is estimated.

1. INTRODUCTION

An electromagnetic wave incident on inhomogeneous isotropic plasmas at the point where its frequency is twice the local Langmuir frequency can decay into two electrostatic plasma waves possessing close frequencies. This nonlinear phenomenon referred to as the two-plasmon parametric decay instability (TPDI) [1-3] is of interest in the laser fusion as a potential source of hot electrons [4] and light at harmonics of half the pump frequency, which may serve for coronal temperature diagnostic [5,6]. A while ago it was shown [7-10] that in the presence of a non-monotonic plasma density profile the absolute TPDI can also occur in the several hundred-kilowatt second harmonic extraordinary (X2) mode ECRH experiments in toroidal magnetic fusion devices (UH). Its excitation entails the low-power-threshold generation of upper hybrid UH waves, at least one of which is localized in the vicinity of a local maximum of the plasma density profile. The latter is often observed at on-axis ECR heating due to the electron-pump out effect or in the presence of the magnetic island.

The TPDI can play an important role in the microwave power absorption. The anomalous absorption rate and distortions of power deposition profile are determined by the instability saturation level. It was investigated recently in the case when the localization of both plasmons is possible, which possesses the lowest threshold and the largest growth rate. A couple of presumably important nonlinear mechanisms responsible for the saturation of the primary TPDI such as a cascade of the secondary decays of primary UH waves [14, 15] and the pump wave depletion [16] were taken into account. The level of the anomalous absorption rate obtained in [11, 16] ranges from 25% up to 40% depending on the density profile peculiarities in the O-point of the magnetic island [17, 18]. The developed TPDI saturation model also allows to explain [10,11] the experimental observations of mysterious anomalous scattering of the pump beam in the X2-mode ECRH experiments at TEXTOR [12, 13], i.e. reproducing the frequency spectrum structure and the spectral power density of a backscattering signal. This provides a reliable basement to the predictions of the theoretical model [11,16] on the high anomalous absorption rate.

However, it should be mentioned that the anomalous backscattering effect was observed at TEXTOR in a wide range of the plasma densities in a local maximum of the density profile substantially exceeding the UH value for half the pump wave frequency. Under these more general conditions the trapping of both the decay UH plasmons is no longer possible, nevertheless, one of the primary UH daughter waves still can be localized. In this situation excitation of a low-threshold absolute TPDI is also possible, as it was demonstrated in [9].

In the present paper we analyze the saturation of the extraordinary pump wave TPDI under the general conditions when only one of the parametrically driven plasmons is trapped in the vicinity of a local maximum of the density profile, whereas the second one can leave the decay region. We consider the secondary decay instability of the localized UH wave as a moderator of the primary TPDI and clarify its role in the saturation. We also estimate the pump power fraction gained anomalously throughout two-UH-plasmon decay in the general case of the excitation of trapped and running UH waves. The general consideration is accompanied in the paper by the numerical analysis performed under the experimental conditions typical of the off-axis X2-mode ECRH experiments at TEXTOR [13]. The emission of electromagnetic waves at the half pump frequency in the high-field-side and low-field side directions is analyzed and the corresponding power is estimated. It is also shown that the nonlinear coupling of the daughter UH waves with the pump could lead to the measurable level of the plasma emission at the 3/2 harmonic of the pump.
2. PRIMARY PARAMETRIC DECAY INSTABILITY

We consider an extraordinary microwave incident on the plasma and analyze its parametric decay in a region where the density profile is non-monotonic. As in inhomogeneous plasma the decay occurs in a narrow layer we introduce the Cartesian coordinate system \((x, y, z)\) with the local maximum of the density profile located at \(x_{\text{max}}\), the \(x\) axis being directed along the flux coordinate inwards the plasma and \((y, z)\) imitating, accordingly, the coordinates perpendicular to and along the magnetic field line on the magnetic surface. In the ECRH experiments at TEXTOR in which the anomalous backscattering of the X-mode pump was observed [13] in a wide density range (at the averaged plasma density higher than \(3 \times 10^{19} \text{cm}^{-3}\)) the pump wave frequency could be either larger or smaller than twice the minimal upper hybrid frequency associated with magnetic island \(\omega_u < 2\omega_{ii}(x_{\text{max}}) = 2\sqrt{\alpha_{\text{pe}}^2(x_{\text{max}}) + \alpha_{\text{ce}}^2(x_{\text{max}})}\) with \(\alpha_{\text{pe}}\) and \(\alpha_{\text{ce}}\) being the electron cyclotron and electron plasma frequencies. In the latter case only one daughter UH wave could be trapped in the \(x\) direction at the local maximum of the density profile. This case is illustrated for \(\alpha_{\text{pe}} = 2\omega_{ii}(x_{\text{max}})\) in Fig. 1, which depicts the dispersion curves of the trapped UH wave \(q_{tt} = q_{tt}(\alpha_{tt}, x)\), \(\alpha_{tt} = \alpha_{tt}\), \(n = 24\) and the non-trapped UH wave \(\alpha_{\text{pe}} = q_{\text{pe}}(\omega_{\text{pe}}, x)\), \(\alpha_{\text{pe}} = \omega_{\text{pe}}\), \(p = 39\) shown by the dash-dotted curves under the conditions of the TEXTOR discharge (the electron temperature \(T_e = 500 \text{eV}\), the magnetic field at the O-point of a magnetic island \(H = 21500 \text{Gs}\), the pump frequency \(\omega_u / 2\pi = 140 \text{GHz}\)). The tertiary decay in this case should lead to excitation of non-trapped UH wave. Thus, its power threshold cannot be overcome and the cascade of decays abruptly ceases. In Fig. 2 we confirm the possibility to satisfy the decay conditions for primary and secondary parametric decays. As it is seen there, in the points \(x_s\) and \(x'_s\) where the solid curve \(q_{tt}\) intersects with the dashed \(q_{tt} = -k_{\text{pe}}\) and dash-dotted \(q_{tt}' = -q_{tt}\) curves the three-wave resonance conditions \(\Delta = q_{tt} - q_{tt}' + k_{\text{pe}}|_{x_s} = 0\) and \(\Delta' = q^{2}_{tt} - q^{2}_{tt}' - q_{tt}'|_{x'_s} = 0\) hold providing necessary conditions for the decay instability.

In order to describe the cascade of parametric decays depicted in figures 1 and 2 we consider the following equations

\[
\begin{align*}
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\text{eu}}(q, \omega) \exp(iq(r - r') - \omega(t - t')) \, dq \, d\omega &= A_{\text{eu}}(r, t) \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\text{eu}}(q, \omega) \exp(iq(r - r') - \omega(t - t')) \, dq \, d\omega &= A_{\text{eu}}(r, t) \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\text{eu}}(q, \omega) \exp(iq(r - r') - \omega(t - t')) \, dq \, d\omega &= A_{\text{eu}}(r, t) \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{\text{eu}}(q, \omega) \exp(iq(r - r') - \omega(t - t')) \, dq \, d\omega &= A_{\text{eu}}(r, t)
\end{align*}
\]

(1)

where

\[
D_{\text{eu}}(q, \omega) = l'_i q'_i + \varepsilon(\omega) q'_i + \frac{\omega_{\text{pe}}^2}{c^2} g(\omega)^2 + \eta(\omega) q_i^4
\]

(2)

are the UH and IB wave’s dispersion functions [19], \(q_i, q'_i\) are the wavenumber components perpendicular to and along the magnetic field, \(\alpha_{\text{pe}}\) and \(\alpha_{\text{ce}}\) are the Langmuir and cyclotron frequencies of the ions and electrons, and \(u_{\text{th}}\) are their thermal velocities, \(l'_i\) is the modified Bessel function of the first kind and the plasma dielectric tensor. The nonlinear charge densities in (1) describing the three-wave coupling are given [20] in the Fourier representation by the following expressions

\[
4\pi \rho_k = \frac{\kappa^2 q_i^4}{H^2} 4\pi \rho_k = \frac{\kappa^2}{H^2} \left( \frac{E_{\text{pe}}}{E_{\text{pe}}} - \frac{\alpha_{\text{pe}}}{\alpha_{\text{ce}}} \right) \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} 4\pi \rho_k' = \frac{\alpha_{\text{pe}}}{\alpha_{\text{ce}}} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} 4\pi \rho_k'' = \frac{\alpha_{\text{pe}}}{\alpha_{\text{ce}}} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H}
\]

in the Fourier representation by the following expressions

\[
4\pi \rho_k = \kappa^2 q_i^4 \frac{E_{\text{pe}}}{H^2} 4\pi \rho_k = \kappa^2 q_i^4 \frac{E_{\text{pe}}}{H^2} - \frac{\alpha_{\text{pe}}}{\alpha_{\text{ce}}} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} 4\pi \rho_k' = \frac{\alpha_{\text{pe}}}{\alpha_{\text{ce}}} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} 4\pi \rho_k'' = \frac{\alpha_{\text{pe}}}{\alpha_{\text{ce}}} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H} \frac{q_k}{H}
\]
FIG. 1. The dispersion curves of daughter waves generated in the course of a cascade of parametric decays; 1 – primary non-trapped UH wave, 2 – primary trapped UH wave, 1’ – IB wave, 2’ – secondary trapped UH waves. The profile of $f_{1,2}^{'}/(f_{2}^{'}/2)$ is given by the thick solid curve.

FIG. 2. The dispersion curves illustrating the possibility of primary and secondary parametric decays. In the points where the solid curve $q_{1,2}$ intersects with the dashed $q_{1,2}$ and dash-dotted $q_{1,2}$ ones the decay resonance conditions hold. The $f_{1,2}^{'}/(f_{2}^{'}/2)$ profile is given by the thick solid curve.

where $E_{0} = \sqrt{16P_{0}^{2}/\nu_{0}(x)}d\exp \left( i\int k_{0}(x_{0},x)dx' - i\alpha_{0}t - \frac{y_{0}^{2}}{2d^{2}} + \frac{z_{0}^{2}}{2d^{2}} \right) + c.c.$ is the pump amplitude, $d$ is the pump beam width, $\nu_{0}$ is the pump wave group velocity, $k_{0}$, $q_{1,2}$, and $q_{1,2}'$ are the wavenumbers of the corresponding Fourier components of fields $E_{0}$, $q_{1,2}$, $q_{1,2}'$ under consideration and the following inequality is assumed $q_{1,2} >> q_{1,2}'$.

We treat the coupled integral equations (1) utilizing the perturbation procedure developed in [21]. In the first step of it we neglect the nonlinear wave interaction and solve homogeneous equations (1) by means of the WKB technique

$$\phi_{1} = \frac{C_{1}}{2D_{1}(x)} \exp \left( i\int q_{1}(x_{0},x')dx' - i\alpha_{1}t \right) + c.c.$$  \hspace{1cm} (3)

where all the amplitudes are constant, $q_{1,2} = q_{1}(x_{0},x')$, $q_{1} = q_{1}(x_{0})$ and $q_{1}' = q_{1}(x_{0})$ are the roots of the UH and IB waves’ local dispersion relations $D_{1}(x_{0}) = 0$ and $D_{2}(x_{0}) = 0$ at $q_{1,2} = 0$,

$$\phi_{n}(x) = \frac{1}{\sqrt{L(x)}} \exp \left( i\int q_{n}(x_{0},x')dx' - i\frac{\pi}{4} \right) + \frac{1}{\sqrt{L(x)}} \exp \left( i\int q_{n}(x_{0},x')dx' - i\frac{\pi}{4} \right).$$  \hspace{1cm} (4)

are the eigenfunction, $\alpha_{n} = \alpha_{1}$, $\alpha_{n}' = \alpha_{1}'$, $\alpha_{n}'$ are the eigen-frequencies obeying the quantization condition $\int_{n}^{n'} q_{n}(x_{0},x')dx' + \int_{n}^{n'} q_{n}(x_{0},x')dx' = \pi(2m+1)$; $m = n,p$; \((x_{0},x_{0}')\) are the UH waves turning points in the radial direction; $D_{2}(x) = \partial D_{2}(x_{0})/\partial q_{n}(x_{0}) = D_{2}(q_{n}(x_{0})/D_{1}(q_{n}(x_{0})))$. The choice $q_{1,2} = 0$ allows reducing the energy losses of the waves from the decay layers.

In the second step of the perturbation procedure we take into account the nonlinear coupling leading to variation of the wave amplitudes. Using the envelope-function approximation we get for amplitudes of waves escaping the decay layer along the $x$ direction

$$C_{1}(r) = -i\frac{1}{\sqrt{D_{1}(x',q_{1}(x'))}} \exp \left( -i\int q_{1}(x_{0})d\xi \right)$$  \hspace{1cm} (5)
Substituting (5) into the second and fourth equations of (1), multiplying the first of them by $\phi_n(x)$ and the second one by $\phi_p(x)$ and integrating over the coordinate $x$ yields

$$
\left( \frac{\partial}{\partial t} + i \Lambda_n \frac{\partial}{\partial y} + i \Lambda_p \frac{\partial}{\partial z} \right) a^* = \nu \exp \left( - \frac{y^2}{d^2} - \frac{z^2}{d^2} \right) a^* - v_i \left[ \frac{\partial}{\partial \omega} a^* \right]
$$

(6)

In (6) we have also introduced the dimensionless amplitudes and coupling coefficients

$$
a^{*,*} = C^{*,*} \sqrt{\frac{\omega^{*,*}}{\omega_{0}}} < \left| D_{n,p} \right| > \frac{d^2}{16 \nu},
$$

(7)

$$
\nu = \frac{\kappa^4}{4 \pi^2} \frac{\partial}{\partial \omega} \left[ \left| D_{n,p} \right|^2 \right] > \sum \left| D_n(x) \right|^2 \left| D_p(x') \right|^2 \phi_n(x) \phi_p(x') \exp \left( i \int (k_n(x) - k_p(x')) dx' \right),
$$

(8)

with $D_n = \partial D_{n} / \partial \omega$, $D_p = \partial D_{p} / \partial \omega$, $\Lambda_n = \kappa^2 D_{n} / \left( 2 \delta q_{n,i} \right)$, $\Lambda_p = \kappa^2 D_{p} / \left( 2 \delta q_{p,i} \right)$ describing the effect of the UH wave diffraction over the magnetic surface, $< ... > = \int dx ... \phi_n(x) \phi_p(x)$ being the averaging procedure over the region of the UH wave localization.

3. ANALYTICAL SATURATION TREATMENT

During the early stage of primary TPDI the amplitude of trapped UH wave, though growing exponentially, is not big enough so that the effects of the secondary decay are negligible. In this case the set of equations (6) can be reduced to the equation

$$
\left( \frac{\partial}{\partial t} + i \Lambda_n \frac{\partial}{\partial y} + i \Lambda_p \frac{\partial}{\partial z} \right) a^* = \nu \exp \left( - \frac{y^2}{d^2} - \frac{z^2}{d^2} \right) a^* - v_i \left[ \frac{\partial}{\partial \omega} a^* \right]
$$

(9)

with $\nu$ defined in (8) being constant. This differential equation describes the eigenfunction $a^* \propto \psi_\xi (y) \psi_\omega (z)$, $\psi_\xi (\xi) = \exp \left( - \frac{\xi^2}{2 \Lambda_n d} \right) H_n \left( \frac{\xi}{\sqrt{\Lambda_n d}} \right)$ with the growth rate $\nu^{*,m} = \nu - \cos \left( \frac{\arg \nu - \pi}{2} \right) \sqrt{\left( 2k + 1 \right) \Lambda_n d / \left( 2m + 1 \right) \Lambda_p d}$.

(10)

The power threshold $P_0^{th}$ of the TPDI is a solution of equation $\nu^{*,m} = 0$. For the most dangerous fundamental mode $k,m = 0$ and under the conditions used earlier in figure 1 it is equal to $P_0^{th} = 127.8$ kW.

The secondary decay of UH wave leads to the saturation of the primary instability. The interplay of the pump wave depletion and the secondary decay of UH wave leads to saturation of the primary instability. The averaged density of the primary plasmons energy $< P_{\xi \omega} >_{pdi}$ in this regime can be estimated from the second equation in the set of equations (6). Namely, the growth rate $\nu$, $< P_{\xi \omega} >_{pdi}$ of this decay should be compensated by the secondary UH plasmon diffractive loss rate from a spot of the pump beam. The latter is characterized by a typical time $\tau_d = \pi d^2 / \max \left( \Lambda_n d, \Lambda_p d \right)$. The corresponding primary plasmons energy density level within the pump wave spot is given by

$$
< P_{\xi \omega} >_{pdi} = \frac{1}{\tau_d \nu}.
$$

(11)

The averaging procedure $< ... >_{pdi}$ here is defined for an arbitrary function $F$ in the following way

$$
< F >_{pdi} = \left( d^2 \right)^{-1} \int dydz F(y,z) \exp \left( - y^2 / d^2 - z^2 / d^2 \right).
$$

The averaged density of the secondary plasmon energy $< a_n^* >_{pdi}$ saturates at a level balancing the nonlinear pumping rate of primary UH wave and its loss rate due to the secondary decay. This saturation level is given by
In the next section the general analytical consideration is accompanied by the numerical analysis performed for the experimental conditions typical of the off-axis X2-mode ECRH experiments at TEXTOR in which the anomalous scattering of the pump wave was observed.

4. NUMERICAL SATURATION TREATMENT

The results of numerical modeling for the incident pump power $P_\text{p} = 600$ kW are shown in Fig. 3 and Fig. 4. In Fig. 3 we demonstrate the temporal evolution of the averaged density of the primary (solid curve) and secondary (dashed curve) UH plasmon energy given in the logarithmic scale. In the early stage of primary TPDI lasting approximately up to $1.2 \mu s$ the growth of the energy density of primary UH plasmons plotted in semi-logarithmic scale is perfectly described by the analytical dependence $2\nu^{0,0}t$ shown by the dash-dotted line with $\nu^{0,0}$ being determined in (10). It should be underlined that due to the losses of the non-trapped UH wave from the decay region the growth rate of TPDI obtained here for the case of only one trapped UH wave is smaller than the growth rate predicted in [11,16] for the two trapped UH plasmon case. Nevertheless, it is large enough to exclude the instability quasi-linear saturation (for example by the magnetic island rotation). In the stationary regime we can see a reasonable agreement of analytically predicted saturation levels (11) and (12) shown by the horizontal dash-dotted lines with the results of numerical modelling. In Fig. 4 we illustrate the temporal relaxation of the amount of power lost by the pump $\alpha = \Delta P / P_\text{p}$ (anomalous absorption rate) when approaching the saturation regime. As one can see when the transition to the saturation regime is managed by the secondary decay of primary wave up to 10% of the pump power is absorbed anomalously.

This value being smaller than the anomalous absorption rate predicted in the case of two trapped plasmons [11,16], nevertheless is quite significant. In Fig. 5 we demonstrate the dependence of the anomalous absorption rate on the pump power. It should be stressed here that the anomalous absorption rate is growing with the pump power close to the TPDI threshold. The growth is saturated at the level of $\alpha = \Delta P / P_\text{p} = 0.11$. Unfortunately, the ECRH power deposition profiles were not measured in the TEXTOR experiments where the anomalous backscattering effect was observed and investigated in detail [12, 13]. Thus the direct comparison of the present paper predictions to the experimental results is not possible. However the model of the two-UH-plasmon decay, utilized in the present paper for the case of only one trapped UH plasmon, in the case of two trapped plasmons (see, for example [11]) had allowed reproducing the fine structure of a frequency spectrum and the spectral

$$\langle |\rho_{\text{pr}}^t| \rangle^2 \cong \frac{v^t}{v_s} \quad (12)$$
power density of a backscattering signal both measured experimentally [13]. The physical mechanism used in [11] for explanation of the backscattering signal generation was based on the nonlinear coupling of the decay plasmons. It is quite natural to assume that the same mechanism could be responsible for producing the backscattering signal also in the case, when only one decay UH plasmon is trapped. Moreover at a similar or slightly lower level of plasmon amplitude predicted in the present paper one should expect a similar or slightly lower level of the anomalous backscattering signal. These expectations appear to be in a qualitative agreement to the TEXTOR experiment [12, 13], where the anomalous backscattering effect was observed at the similar, or lower power level in the wide density range at plasma densities not allowing trapping of the both decay plasmons.

5. MICROWAVE EMISSION AT THE PUMP SUB-HARMONICS

It should be also underlined that in the case $\omega_b = 2\omega_{\text{in}}(x_{\text{m}})$ considered in this paper the non-trapped UH wave propagating perpendicular to the magnetic field outward is converted into the X-mode (see Fig. 1). It crosses the ECR surface and leaves the plasma at the high-field side. Neglecting ECR absorption of the X-mode, which is small at the perpendicular propagation in modest temperature plasma of middle scale devices [26], and taking into account the power balance in the decay we can estimate the power of this X-mode radiation at the half-pump frequency as $P_{\text{XPP}} = \alpha P_b / 2$. In the case considered in this paper it results in 0.05 $P_b$. The larger part of this power is reflected from the device wall in the form of the X-mode and finally absorbed after conversion in the UHR. However a smaller part of the power, characterized by cross-polarization factor $\beta_{\text{XO}}$, is reflected in the form of O-mode, which is partly absorbed in the ECR and then leaves the plasma. Taking into account the O-mode absorption rate [22]

$$\Gamma_{\text{oii}} = \frac{\pi}{2 m_e c^2} \frac{\alpha_{\text{o}} R_{\text{o}}}{N} \left( \frac{\alpha_{\text{o}}}{\alpha_{\text{n}}} \right) \left( \frac{\alpha_{\text{o}}}{\alpha_{\text{n}}} \right) \left( \frac{\alpha_{\text{o}}}{\alpha_{\text{n}}} \right),$$

we obtain the estimation for the O-mode emission power at the low field side at the half pump frequency $Q_o \approx 0.5 \alpha P_{\text{XO}} \exp(-\Gamma_{\text{oii}}) P_b$ (13)

Taking into account that for the conditions of the TEXTOR off-axis ECRH experiment $\Gamma_{\text{oii}} = 2$, and assuming $\beta_{\text{XO}} = 0.01$ we get for the case under consideration $Q_o \approx 8 \times 10^{-4} P_b$. Thus at the maximal microwave power of 600 kW one can expect to see at the low field side of the tokamak the O-mode emission of 50 W at the half pump frequency. This emission is observable only in the narrow density range when the UH density for half the pump frequency is slightly lower than plasma density in the profile local minimum, however the decay with the upper branch of the non-localized UH wave (electron Bernstein wave) is still possible.

Out of this density range the intensive emission of sub-harmonic $\omega_b / 2$ is not possible; nevertheless due to the nonlinear coupling of UH plasmons and the pump wave emission of the $3\omega_b / 2$ harmonic can occur similar to laser fusion experiments [5]. This possibility is illustrated in Fig. 6 where the dispersion curve of primary trapped UH wave (solid line) along with $k_{\text{in}} + \omega_{\text{in}} / c$, $\omega_{\text{n}} = \omega_{\text{b}} + \omega^* \approx 3/2 \omega_{\text{b}}$ (dashed line) is shown. In points where these lines intersect the Bragg scattering condition is fulfilled and a generation of the $3\omega_{\text{b}} / 2$ harmonic takes place. It happens when the following conditions hold

$$\omega_{\text{n}}(x_{\text{m}}) \left| \frac{\delta n_e}{\delta x} \right| < \frac{5\omega_{\text{b}}}{2} \frac{c}{l},$$

where $\delta n_e = n_e(x_{\text{m}}) - \bar{n}_e(x_{\text{m}})$, $n_e(x_{\text{m}})$ and $\bar{n}_e(x_{\text{m}})$ stand for the density value in the local maximum and minimum of the profile accordingly (see Fig. 6). It should be underlined that these inequalities may hold also in the case when both decay plasmons are trapped. In the case under consideration in the present paper only the primary UH wave appears to be able generating the wave at frequency $\omega_{\text{b}} + \omega^*$ propagating outwards. The microwave signal at frequency $\omega_{\text{b}} + \omega^*$ received by an antenna on the low-magnetic field-side of the toroidal device can be calculated with the help of reciprocity theorem [23] in the following form.

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**FIG. 6.** The dispersion curves of primary trapped UH wave (solid line) generated in the course of a cascade of parametric decays. In the points where the curves intersect the nonlinear coupling is possible. The dashed line gives $k_{\text{in}} + \omega_{\text{in}} / c$. The density profile in the magnetic island is given by the thick solid curve. Its local maximum corresponds to the O-point of a magnetic island.
\[ A(\omega_s) = \frac{1}{4} \int J_\omega (\omega_s, r) E^\ast (\omega_s, r) d\mathbf{r}, \]  \hspace{1cm} (15) \]

The integration in (15) is carried out over the whole plasma volume and the normalized to unite power receiving antenna beam \( E^\ast (\omega_s, r) \) with the waist \( d_\omega \) in a vicinity of the O-point of the magnetic island is defined as follows:

\[ E^\ast = e^{-\frac{1}{c} \frac{8}{d_\omega^2} \exp \left( -\frac{y^2 + z^2}{2d_\omega^2} \right)} \left[ \exp \left( -i \frac{\omega_s}{c} (x - a) \right) + \text{c.c.} \right], \]

where we have assumed the magnetic island in the near-wave zone of receiving antenna. The nonlinear current describing the excitation of the X-mode at frequency \( 3\omega_b/2 \) being generated due to the interaction between the primary UH wave and the pump wave is given by the expression

\[ (i_s (\omega_s, r) e_s) = -\frac{i}{4\pi} \frac{\omega_s - \omega_0}{\alpha_0^2 - \omega_0^2} \frac{E_{oe}}{H}, \]

where we have retained the dominant terms keeping in mind that \( \omega_s \approx \omega_{0}\), and \( (\omega_s + \omega_0)^2 \alpha_0^2 >> \omega_s^2 \). The backscattering signal power \( p_s \) equal to \( |A(\omega_s)|^2 \) can be estimated based on (15) using the stationary phase method as

\[ p_s = \frac{1}{2\pi} \frac{\omega_s^2 \omega_{0e}^2}{\omega_0^2 \omega_0^2} \frac{8T_s}{\pi} \left( \omega_s^2 x_s \right) \left( d_\omega^2 + d_{\alpha}^2 \right) H^2 \frac{q_\alpha^2 (x_s) \cos \phi_s}{\pi} \frac{1}{\tau_{\text{pdi}}^2} \frac{1}{P_0^2}, \]

where \( x_s \) stands for the stationary phase or the scattering point where the \( 3\omega_b/2 \) emission is generated and we have introduced the scattering coherence length

\[ \tau_s = \int \frac{d\phi_s(x) \exp \left( i \frac{\omega_s + \omega_0}{c} (x - a) \right)}{L_s (\omega_s, x)}. \]

Thus, we get the \( 3\omega_b/2 \) pump harmonic emission power received by antenna as \( p_s = T_{3/2} P_0 \), where the conversion coefficient is given by

\[ T_{3/2} = \frac{1}{2\pi} \frac{\omega_s^2 \omega_{0e}^2}{\omega_0^2 \omega_0^2} \frac{8T_s}{\pi} \left( \omega_s^2 x_s \right) \left( d_\omega^2 + d_{\alpha}^2 \right) H^2 \frac{q_\alpha^2 (x_s) \cos \phi_s}{\pi} \frac{1}{\tau_{\text{pdi}}^2} \frac{1}{P_0^2}. \]

Under the plasma parameters considered in the paper and for \( d_\omega = 2d \) we get \( T_{3/2} = 5 \cdot 10^{-5} \). For the maximal power utilized in the TEXTOR magnetic island control experiment it results in 30 W of received \( 3\omega_b/2 \) pump harmonic emission.

6. CONCLUSIONS

We have analyzed the non-linear stage of the extraordinary pump wave parametric decay into two UH waves in the case when only one of them is localized along the direction of inhomogeneity. Considering the secondary low-threshold decay instability as the most likely scenario of the primary TPDI saturation, we have derived a set of reduced equations describing the decay instability growth and saturation, estimated the level of parametrically driven UH wave. The general analytical consideration was accompanied by the numerical analysis performed for the experimental conditions usual for the off-axis X2-mode ECRH experiments at TEXTOR in which the anomalous scattering of the pump wave had been observed. The instability threshold of 128 kW was determined for specific parameters of the computation and decay waves. The pump power fraction absorbed anomalously was estimated as 10%.

Emission of the pump wave sub-harmonics was analyzed. The power of the plasma microwave emission at the pump wave half frequency \( \omega_b/2 \) was estimated at the ECRH power 600 kW and shown to be huge (30 kW) at the high field side in the extraordinary polarization and easily measurable (50 W) at the low field side in the ordinary polarization. The microwave emission signal in extraordinary polarization at the low field side at the frequency \( 3\omega_b/2 \) was also estimated and shown to be sufficiently large for registration (30 W) as well.

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