Explanation of prompt growth of ECE signal in tokamak runaway electron experiments

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Motivation: Non-thermal ECE signal observed in runaway electron experiments in tokamaks

- In both quiescent runaway electron experiments (QRE) in DIII-D, and in disruption experiments in TFTR, fast growth of ECE signal has been observed.

C. Paz-Soldan et al., Nucl. Fusion 56, 056010 (2016).
Motivation: Non-thermal ECE signal observed with runaway electron experiments in tokamak (cont’d)

- The non-thermal ECE signal comes from runaway electron (RE) in plasma.

- The growth rate of ECE signal is much higher than HXR growth rate.
- The contribution to high harmonics is more prompt than low harmonics.
Why can the RE produce such strong ECE signal?

- Though highly energetic, the number of runaway electrons is still very small compared to thermal electrons.

- Pitch angle of RE is typically very small. Most of the energy is in parallel direction.

To answer this question, we need to develop an ECE synthetic diagnostic tool for runaway electrons.
Explanation of nonthermal ECE signals

- Emission and absorption of electron cyclotron waves from runaway electrons.
  - The high energy RE tail has strong emission but little absorption of electrons cyclotron waves.
  - Unlike the thermal electrons, the contribution to ECE from RE is nonlocal.

- Kinetic instabilities associated with runaway electrons
  - Unstable whistler waves can cause significant increase of the pitch angle of RE, which leads to strong ECE.
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Absorption and emission of electron cyclotron waves

- The absorption and emission of electron cyclotron wave are associated with electrons satisfying the ECE resonance condition

\[ \omega - k_v - \frac{n \omega_{ce}}{\gamma} = 0 \]

- The absorption of the ECE can be represented by \( \epsilon^A \) (anti-Hermitian part of permittivity tensor)

\[ k_i = -\frac{\mathbf{E} \cdot \omega^2 \epsilon^A / c^2 \cdot \mathbf{E}}{\mathbf{E} \cdot \partial / \partial k (\mathbf{kk} - k^2 + \omega^2 \epsilon / c^2) \cdot \mathbf{E}} \]

- The emission of ECE can be represented by source current correlation tensor \( K \)

\[ K_{\omega,k} 4\pi \delta(\omega - \omega') \delta(k - k') = \langle \mathbf{j}_{\omega,k} \mathbf{j}_{\omega',k'} \rangle \]

- \( K \) depends on the value of the distribution function \( f \) at the resonance regions, whereas \( \epsilon^A \) depends on the gradients of \( f \) (with respect to \( p \) and \( \xi \)) in the resonance regions.
Electron cyclotron wave absorption from thermal electrons is strongly localized

Here we show one example of calculation of electron cyclotron wave absorption ($k_i$) for $\omega=2.5\omega_{ce}(R_0)$ and a Maxwellian electron distribution ($T_e=1.8$ keV, constant in space)

- High harmonic absorption is significantly lower than low harmonic.
- The absorption is localized near the resonance region.

- Plasma is optically thick for $n=2$ resonance, whereas optically thin for $n=3$ resonance.
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- Ratio of emission to absorption is a constant and proportional to $T_e$. 

Electron cyclotron wave absorption & emission from thermal electrons are strongly localized
Runaway electron distribution in momentum space

- RE distribution evolution in momentum space is calculated using a Finite Element code solving the kinetic equation with time.
  - Include the secondary RE generation (runaway electron avalanche)
  - Include the synchrotron radiation energy damping

\[ n_e = 0.6 \times 10^{19} \text{ m}^{-3}, T_e = 1.8 \text{keV} \]

\[ E = 0.055 \text{ V/m}, \frac{E}{E_{CH}} = 9 \]

\[ B = 1.5 \text{T} \]

- The calculated RE tail is flat in \( p \) at \( \xi = 1 \), but strongly anisotropic in pitch angle
**ECE absorption from RE is insignificant**

Here we show one example of calculation of electron cyclotron wave absorption \( (k_i) \) for \( \omega = 2.5\omega_{ce}(R_0) \), with RE tail distribution.

- RE contribution to the absorption is nonlocal and not limited to resonance region of thermal electrons
- The runaway electrons have little impact on the ECE wave absorption.
ECE emission from RE is important and accessible

Here we show one example of calculation of electron cyclotron wave absorption ($k_i$) and emission ($\mathbf{E} \cdot \mathbf{K} \cdot \mathbf{E}$) for $\omega=2.5\omega_{ce}(R_0)$, with RE tail distribution.

- The emission-absorption ratio of ECE from RE is much larger than thermal electrons.
- RE distribution is not in thermal equilibrium.
- Given plasma is optically thin for high harmonics ($n \geq 3$) electron cyclotron wave, the radiation from RE can propagate outside and get collected by receiver.
ECE from RE at high frequency has less absorption from thermal electrons

- If the ECE is dominated by RE, then for high frequency ECE, the radiation power can be stronger.
- The RE ECE at high-field-side can propagate to low-field-side with little absorption.
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Electron diffusion in momentum space through wave particle interaction

• The electrons can be diffused in momentum space by the electromagnetic waves in the plasma, if satisfying resonance condition.

• For Cherenkov resonance ($\omega - k_{\parallel} v_{\parallel} = 0$), diffusion happens in the parallel direction.

• For Doppler resonance ($\omega - k_{\parallel} v_{\parallel} = n\Omega_e$), wave causes both energy diffusion and pitch angle scattering.
  • For anomalous Doppler resonance ($n<0$), particles transfer energy to the waves when they get isotropized by the wave diffusion.
Framework of hybrid simulation of RE distribution function and whistler mode amplitudes

\[ f_{RE} \Rightarrow \begin{cases} \text{Growth Rates} \\ \text{Fluctuation Amplitudes} \end{cases} \]

Coefficients for quasilinear diffusion operator
Whistler mode amplitudes at $t=0.10s$

**RE distribution at $t=0.10s$**

$f_{RE}(p, \xi=1)$ at different time
**RE can be strongly scattered by excited whistler modes**

- When RE population reaches a threshold, certain whistler modes become unstable and can grow to large amplitudes
  - The growth rate is faster than RE population growth
- Excited whistlers cause strong pitch angle scattering of the runaway electrons.
  - It affects RE in both low and high energy regimes
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Reciprocity method: a faster way to calculate ECE signal power

• Instead of solving the radiated power $P$ by collecting waves from all the sources, we solve the reciprocal problem, and calculate $P$ using reciprocity theorem.

• Reciprocal Problem:
  • Reversed in time
  • Study the transposed plasma with $\varepsilon(\omega; r, r') = \varepsilon^T(\omega; r', r)$
  • Calculate $E^+$ of the wave that launched from the receiver, propagates into plasma, absorbed by resonant electrons.

• The measured radiation power

$$P(\omega) = \frac{1}{8\pi} \int d\bar{x} \tilde{E}^+_{\omega,k} \cdot K \cdot \tilde{E}^{+*}_{\omega,k}$$

Runaway electron ECE signals from synthetic diagnostic agrees with experiments qualitatively

- ECE signals from synthetic diagnostics shows exponential growth and exceeds thermal electron temperature as RE population grows.

- The catch-up of higher harmonic signal is observed.

- The growth rate is still smaller compared to experiments.

![ECE signals from synthetic diagnostic](image)
Spectrum of ECE signal with RE is smoother than thermal electrons

- For thermal electrons, the radiation power spectrum is like a step function.

- Growth of RE makes the spectrum flatter, which is consistent with the experiments.
Summary

• Using newly-developed RE ECE synthetic diagnostic tool, we successfully calculate the ECE signals from runaway electrons and benchmark with experiments.
  • Due to the small absorption and large emission, and nonlocal spatial distribution of both, runaway electrons can give important contributions to ECE radiation.
  • Excited whistler modes can significantly increase the pitch angles of runaway electrons and enhance the ECE signals.

• Future plans:
  • Use realistic profiles of plasma parameters
  • Develop more synthetic diagnostic tools for RE (HXR, Gamma-ray, Synchrotron Emission)
  • Integrate the module to SDP (Synthetic Diagnostic Platform) developed at PPPL