Validation of MHD Models using MST RFP Plasmas


2nd IAEA Technical Meeting on Fusion Data Processing, Validation and Analysis
Cambridge, Massachusetts • May 30th – June 2nd, 2017

This material is based upon work supported by the U.S. Department of Energy Office of Science, Office of Fusion Energy Sciences program under Award Number DE-FC02-05ER54814. This material is based upon work supported by the National Science Foundation under Grant No. PHY 08-21899.
• Introduction to Validation and MST

• Single-fluid MHD Validation

• Two-fluid MHD Validation

• Maximizing the Utility of Diagnostic Measurements

• Conclusions and Future Work
Validation is vital to fusion energy programs across the world

- Computational models are improving, but how do we know if their predictions are correct?

- **Validation**: quantitative assessment of the degree to which a computational model is an accurate representation of the real world
  - Asks if simulation agrees with experiment under a rigorous comparison
  - This is *not* simply plotting simulation results against experimental results
  - Quantitative, inclusion uncertainty is a key consideration
  - Allows *objective* comparisons of conceptual models

- Validation is a significant challenge
  - The effort requires *substantial* resources (physicists, dedicated experiments, computational resources)
  - Validation is still in the exploratory phase in plasma physics
  - The community must evaluate what it means to validate
  - Validation is more routinely performed in computational fluid dynamics and aerospace industry

Greenwald, Phys. Plasmas 17, 058010 (2010)
Validation metrics allow quantitative comparisons between simulation and experiment with consideration of uncertainty

• Validation metrics can take on various forms

\[ M_j = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{x_i - y_i}{y_i} \right| \]

\[ M_j = 1 - \frac{1}{n} \sum_{i=1}^{n} \text{tanh} \left( \left| \frac{x_i - y_i}{y_i} \right| + \left| \frac{\sigma_{y_i}}{y_i} \right| + \left| \frac{\sigma_{x_i}}{x_i} \right| \right) \]

\[ M_j = \frac{1}{N_{\text{degrees}}} \sum_{i=1}^{n} \left( \frac{x_i - y_i}{\sigma_{y_i} + \sigma_{y_i}} \right)^2 \]

\[ \rightarrow \begin{cases} 0 \quad & \text{agreement} \\ \infty \quad & \text{disagreement} \end{cases} \]

\[ \rightarrow \begin{cases} 0 \quad & \text{disagreement} \\ 1 \quad & \text{agreement} \end{cases} \]

\[ \rightarrow \begin{cases} 0 \quad & \text{perfect agreement} \\ 1 \quad & \text{expected agreement} \\ \infty \quad & \text{disagreement} \end{cases} \]

• There are many possible metrics
  – Metric must identify salient elements of the model being tested
  – Metric must confront disagreement between simulation and experiment
  – Metric should incorporate uncertainties in both experiment and simulation
Comparison of many quantities is essential to meaningfully compare simulation and experiment

- Primacy hierarchy tracks how quantities combine to produce other quantities
  
  ![Diagram showing Primacy levels for simulation](P)

  1. \(\tilde{V}\)
  2. \(\langle \tilde{V} \times \tilde{B} \rangle\)
  3. \(E\)
  4. Sawtooth Period

  Equilibrium profiles

  \(q\)

- Composite metrics characterize overall agreement of entire simulations

\[
V = \sum_{j}^{n} M_j P_j S_j W_j \\
S: \text{sensitivity rating} \\
W: \text{repetition weight}
\]

Validation benefits from using multiple magnetic configurations with a large parameter space

- RFP provides complimentary parameter space to tokamaks
  - RFP: \( q < 1 \) everywhere, tokamak: \( q \geq 1 \) typically
  - Multiple tearing modes interact nonlinearly, rich test of physics
  - Successful validation in both RFP and tokamak configurations increases confidence in the predictive ability of a model

- MST in particular is well-suited for validation
  - Plasmas are well-diagnosed
    - Magnetic Probes, TS, CHERS, FIR
      Interferometry/Polarimetry, SXR, HXR, Rutherford Scattering
  - Plasmas are well-controlled
    - New programmable power supply (PPS) configuration
    - Allows RFP plasmas with \( I_p \) from 60 kA to 600 kA
    - Increased flat top time to up to 65 ms

\[ R_0 = 1.5 \text{ m} \]
\[ a = 0.52 \text{ m} \]
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Primary goal is to validate MHD models, focusing on how magnetic fluctuations at the edge $\tilde{b}_a$ scale with Lundquist number $S$.

- Dominant thermal energy transport mechanism is based on interaction of MHD tearing fluctuations:

$$\chi_{RR} \sim \tilde{b}_{m,n}^2$$

- $S$ must increase for a reactor, so understanding how $\tilde{b}$ scales with $S$ is key:

$$S = \frac{\tau_R}{\tau_A} = \frac{\mu_0 a^2}{\eta(Z_{eff}, n_e, T_e)} \cdot \frac{B}{a\sqrt{\mu_0 \rho}} \sim T_e^{3/2} I_P$$

### Data Analysis

- Integrated data analysis
- Interferometry
- Thomson scattering
- Magnetics

$$\tilde{b}_a = \alpha \cdot S^\beta$$

$$\frac{B_{P,a,m,n}}{\langle B_{P,a,0,0} \rangle}$$

Value on axis
Experimental $T_e$ and $S$ span large range

- First $T_e$ measurements in ultra-low current plasmas
  - $40 \text{ eV} \leq T_e \leq 400 \text{ eV}$
  - $4 \times 10^4 \leq S \leq 9 \times 10^6$
Two MHD codes are used to simulate MST RFP plasmas

- **DEBS**
  - Nonlinear, single-fluid, visco-resistive, MHD code
  - Restricted to cylindrical geometry
  - Historically used in RFP community, can also be used for tokamak simulations

\[
\frac{\partial \mathbf{A}}{\partial t} = \mathbf{S} \mathbf{V} \times \mathbf{B} - \eta \mathbf{J}
\]

\[
\rho \frac{\partial \mathbf{V}}{\partial t} = -\mathbf{S} \rho \mathbf{V} \cdot \nabla \mathbf{V} + \mathbf{S} \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}
\]

- **NIMROD**
  - Nonlinear, two-fluid, extended MHD code, includes gyroviscous stress terms
  - Flexible geometry
  - Widely used in RFP, tokamak, spheromak, reconnection simulations, and other configurations

\[
\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{en} - \frac{\nabla p_c}{en} + \eta \mathbf{J} + \frac{m_c}{e^2 n} \frac{\partial \mathbf{J}}{\partial t}
\]

\[
m_{in} \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_{gyro} - \nabla \cdot \nu m_{in} \mathbf{W}
\]
Series of nonlinear single-fluid DEBS and NIMROD runs performed to determine $\tilde{b}_a$ scalings

- Both codes run in single-fluid, cylindrical geometry, $Pm = \mu_0 \nu / \eta = 1$, $R_0 / a = 3$
- Initial non-reversed paramagnetic pinch equilibrium unstable to several tearing modes
- Plasma reverses as tearing modes grow in amplitude
- Magnetic fluctuation amplitudes at edge determined during mean of nonlinear saturated state
Series of nonlinear single-fluid DEBS and NIMROD runs performed to determine $\tilde{b}_a$ scalings

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- Initial non-reversed paramagnetic pinch equilibrium unstable to several tearing modes
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- Magnetic fluctuation amplitudes at edge determined during mean of nonlinear saturated state
Magnetic fluctuations at edge fit to power law scaling

\[ \tilde{b}_a = \alpha \cdot S^\beta \]

### Power Law Amplitudes

<table>
<thead>
<tr>
<th>n</th>
<th>MST</th>
<th>NIMROD</th>
<th>DEBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.71 ± 0.264</td>
<td>0.04 ± 0.023</td>
<td>0.30 ± 0.183</td>
</tr>
<tr>
<td>6</td>
<td>0.09 ± 0.029</td>
<td>0.07 ± 0.013</td>
<td>0.10 ± 0.051</td>
</tr>
<tr>
<td>7</td>
<td>0.13 ± 0.031</td>
<td>0.05 ± 0.014</td>
<td>0.05 ± 0.019</td>
</tr>
<tr>
<td>8</td>
<td>0.12 ± 0.036</td>
<td>0.04 ± 0.024</td>
<td>0.09 ± 0.018</td>
</tr>
</tbody>
</table>

### Power Law Scalings

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</tr>
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<tbody>
<tr>
<td>5</td>
<td>−0.39 ± 0.028</td>
<td>−0.27 ± 0.053</td>
<td>−0.45 ± 0.063</td>
</tr>
<tr>
<td>6</td>
<td>−0.20 ± 0.023</td>
<td>−0.20 ± 0.020</td>
<td>−0.25 ± 0.053</td>
</tr>
<tr>
<td>7</td>
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<td>−0.21 ± 0.028</td>
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Simulated magnetic fluctuation amplitudes at edge depend on magnetic Prandtl number $Pm$

- Magnetic Prandtl number represents relative importance of viscous to resistive diffusion
  \[ Pm = \frac{\mu_0 \nu}{\eta} \]
  - Braginskii perpendicular ion viscosity gives $Pm = 0.25$
  - Independent MST experiments suggest $Pm = 55 - 90^{(a)}$ and $Pm = 60 - 280^{(b)}$
  - Recent work indicates $\nu \sim \tilde{b}^2$
  - Anisotropy could be important

- NIMROD $Pm$ scan at $S = 10^4$
  - Results not sensitive to $Pm$ for $Pm < 10$
  - Relative mode strength drastically changes for $10^{-4}$
  - Suggests that globally-measured $Pm$ are not directly related to local Braginskii $Pm$ value

(a) Almagri, Phys. Plasmas 5, 3982 (1998)
(b) Fridström, Phys. Plasmas 23, 062504 (2016)
Two-fluid effects important in both simulation and experiment

Adapted from Sauppe, Phys. Plasmas 24, 056107 (2017)

Triana, Ph.D. Dissertation (2017)
Two-fluid physics can bring simulations into better agreement

\[ S = 2 \times 10^4 \]

**Sauppe NIMROD**

**Previously-shown NIMROD**

**MST**

**Sauppe NIMROD**
Validation motivates careful consideration of diagnostic data

\[ T_e = T_{e0} \left( 1 - \left| \frac{z}{a} \right|^{\alpha} \right) \]

- Suspect spatial channels had issues with instrumental functions
- Recalibration with better calibration light source fixed this issue
- Thomson laser beamline being modified to reduce stray light and allow \( T_e \) and \( n_e \) measurements at all locations
Conclusions

• MST is an excellent test bed for validation
  – RFP is complimentary to tokamaks
  – MST can access wide parameter range, has necessary diagnostics

• Single-fluid MHD simulations show some agreement with MST measurements
  – Scalings are similar
  – Amplitudes can differ quite a bit

• Two-fluid NIMROD simulations can show better agreement with experimental amplitudes

• Validation is a challenge
  – We are only just reaching the point of beginning to evaluate validation metrics
Future Work

- Extensive experimental data collection
  - Improved Thomson scattering diagnostic should be ready later this summer
  - Investigation at even lower $I_p$ and $S$

- Further simulation
  - Single-fluid MHD at higher $S$
  - Two-fluid MHD at several $S$ to allow scaling determination
  - Inclusion of toroidal circuit model in DEBS and NIMROD simulations

- Evaluation of validation metrics for profiles
  - Up to this point focus has been on core and edge quantities
  - This will allow evaluation of composite metrics

- Collaboration with RELAX RFP extends parameter space for validation (Karsten McCollam)
  - Typical $10^4 \leq S \leq 10^5$
  - Aspect ratio $A = R_0/a = 2$ motivates cases with cylindrical and toroidal geometry