Impact of diverted geometry on turbulence and transport barrier formation in 3D global simulations of tokamak edge plasma

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Edge plasma turbulence: an ITER-relevant topic

Turbulent transport of energy and particles

- Impact on confinement time, and possibly on the **transition from Low to High confinement** mode → tokamak performances
- Determines the **Scrape-Off Layer (SOL) width** and **heat flux poloidal asymmetries** → heat flux at target plates

\[ q < 10 \frac{MW}{m^2} \]
Modelling edge plasma turbulence: reducing to 2D

2D Fluid turbulence codes

Examples: TOKAM2D, HESEL ...

Focused on a region with bad curvature, unstable to interchange turbulence:
\[ \vec{v}_p \cdot \vec{v}B > 0 \]

Based on flute assumption

\[ k_\| \ll k_\perp \rightarrow k_\| \approx 0 \]

Averaged parallel dynamics

Advantages:

- Relatively low computational time
- Good agreement with experimental results (e.g. Y. Sarazin et al. PoP 1998)
- Able to reproduce transport barriers (e.g. J.J. Rasmussen PPCF2016) acting on key parameters
Including parallel dynamics

3D codes in circular (limiter) geometry

Examples: GBS (Switzerland), BOUT++ (UK), GEMR (Germany), TOKAM3X (France)

- Put in evidence **poloidal asymmetries** in turbulent transport: \( k_\perp \gg k_\parallel \approx \frac{2\pi}{2\pi qR} > 0 \)
- Qualitatively recover the features of parallel dynamics in the SOL
- Maintain a good description of blob dynamics and statistical properties of turbulence

(P. Tamain et al. PET conference 2013)
**TOKAM3X can handle diverted magnetic configurations**

We need to extend turbulent simulations towards a realistic geometry since:

- **H-mode** experimentally observed preferentially in divertor configuration
- Transverse transport in the divertor region is believed to affect the heat flux profile at the target
- Temperature and density fluctuations in the divertor could affect recycling and sputtering

**Objective:** understanding the effect of complex geometry on turbulent transport
TOKAM3X: 3D flux-driven, fluid turbulence code

**Mass conservation**
\[ \partial_t N + \nabla \cdot \left( (\Gamma - J_\parallel) \vec{b} \right) + \nabla \cdot \left( N(\vec{u}_E + \vec{u}_{\nabla B}^e) \right) = S_N + D_N \nabla^2 \perp N \]

**Parallel momentum conservation**
\[ \partial_t \Gamma + \nabla \cdot \left( \frac{\Gamma^2}{N} \vec{b} \right) + \nabla \cdot \left( \Gamma (\vec{u}_E + \vec{u}_{\nabla B}^i) \right) = -2 \nabla_\parallel N + D_\Gamma \nabla^2 \perp \Gamma \]

**Vorticity conservation**
\[ \partial_t W + \nabla \cdot \left( W \frac{\Gamma}{N} \vec{b} \right) + \nabla \cdot \left( W \vec{u}_E \right) = \nabla \cdot \left( N \vec{u}_{\nabla B}^i - N \vec{u}_{\nabla B}^e \right) + \nabla \cdot \left( J_\parallel \vec{b} \right) + D_W \nabla^2 \perp W \]

**Ohm’s law**
\[ \eta_\parallel N J_\parallel = \nabla_\parallel N - N \nabla_\parallel \Phi \]

**Vorticity definition**
\[ W = \nabla \cdot \left( \frac{1}{B^2} \left( \tilde{\nabla}_\perp \Phi + \frac{\tilde{\nabla}_\perp N}{N} \right) \right) \]

**Drifts**
\[ \vec{u}_E = \frac{\vec{B} \times \tilde{\nabla} \Phi}{B^2} \quad \vec{u}_{\nabla B}^{i/e} = \pm 2 T_{i/e} \frac{\vec{B} \times \tilde{\nabla} B}{B^3} \]

**Boundary conditions**
\[ \frac{\partial}{\partial \psi} \cdot = 0 \]
Periodic in \( \varphi \)

Periodic in \( \theta \) for closed field lines

Here isothermal version, no neutrals

Bohm conditions in \( \parallel \) direction

\[ k_\perp \ll 1/\rho_L \]

\[ \delta \approx \pi/2 \]
**Simulated magnetic geometries**

DIVERTOR

COMPASS – like

Flux expansion

X-point

**LIMITER**

~Same parameters,
Different safety factor

\[
q = \frac{1}{2\pi} \int \frac{B^\phi}{R B^\theta} \, d\theta
\]
Turbulent structure shape varies strongly on a flux surface

Balloon character of turbulence is immediately visible

In this region, turbulent structures become \textit{radially elongated} at the X-point

(J. R. Harrison et al. JNM 2015)
If the **flute assumption** is valid, a turbulent structure must be homogeneous over a flux tube

\[ \partial_{s_{\parallel}}(\vec{B} \cdot d\vec{S}) = 0 \]

Flux tube cross section conserved since \( \vec{V} \cdot \vec{B} = 0 \)

\[ S \propto 1/B \propto R \]

**Flux expansion**

\[ f_x(\theta, \psi) = \frac{\vec{V} \psi(\theta, \psi)_{mp}}{\vec{V} \psi(\theta, \psi)} \]

**Magnetic shear**

\[ s_{cyl} = \frac{r \, dq}{q \, dr} \]

**Focusing on a flux surface**

\[ k_\theta \propto \frac{1}{\delta\theta} \propto \frac{\delta r}{S} \propto f_x/R \]
**Turbulent structures are (almost) field-aligned**

We focus on the deformation on a closed flux surface

We calculate locally:

\[ k_\theta \approx \frac{2\pi}{\lambda} \]

And we compare to the flux tube shape

**To a first order, Flute assumption justified!**
3D shape of turbulent structures

Density and electric potential turbulent structures have a finite length in the parallel direction

$$L_{\parallel} \approx \pi qR$$

Typical extension in parallel direction $\sim \frac{1}{2}$ poloidal turn

$$k_{\parallel} \approx \frac{2\pi}{2\pi qR} \approx \frac{2\pi}{10^4 \rho_L} \ll k_{\perp} \approx \frac{2\pi}{10 \rho_L}$$
Transverse $E \times B$ fluxes are proportional to flux expansion

Macroscopically, the poloidal variation of the structure shape has an impact on fluxes

$$u_E^\psi \approx -\frac{1}{B} \partial_\theta \Phi \sim R k_\theta \Phi \sim f_x u_E^\psi_{|mp}$$

$$k_\theta \approx f_x k_{\theta mp} \frac{R_{mp}}{R}$$

Transverse fluxes **not proportional** to radial pressure gradient

Not easily described by a diffusive approach: $\Gamma^\psi \approx -D(\psi, \theta) \langle \nabla N \rangle^\psi$
Getting rid of geometric effects: a remapping operation

We are interested in transverse transport across the flux surfaces

Normalizing fluxes to the flux expansion: usual picture of a ballooned transport

Still, a peculiar flux pattern is visible in the divertor region
Turbulent and mean-field components of the flux

\[
\left\langle N \cdot u^\psi_{E \times B} \right\rangle_{t,\Phi} = \left\langle N \right\rangle_{t,\Phi} \left\langle u^\psi_{E \times B} \right\rangle_{t,\Phi} + \left\langle \tilde{N} \cdot \tilde{u}^\psi_{E \times B} \right\rangle_{t,\Phi}
\]

Mean-field flux
Already included in 2D transport codes as SOLEDGE2D, SOLPS, EDGE2D, UEDGE..

Turbulent flux
Included in other 3D turbulent codes, but rarely with realistic geometry

(D. Galassi et al. NF 2017)
Significant turbulent transport in the outer divertor leg

Outer divertor leg is **unstable** $\vec{v}_p \cdot \vec{v}_B > 0$

Turbulent transport **towards the far SOL**

$\sim 20\%$ midplane

Potentially interesting for the **spreading of $\lambda_q$**

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![Average density profile at the outer target](image)

**Average density profile at the outer target**

- **$\langle N_\psi \rangle_{t,\phi}$ (a. u.)**
- **Z ($\rho_L$)**
- **R ($\rho_L$)**

**$<\tilde{N}\psi>_t/f_x$ (a. u.)**

X $10^{-3}$

20

15

10

5

0

-5

600 700 800 900

Broad density profile at the outer target
Fluctuations are **damped in the vicinity of the X-point** (turbulent transport inhibited)

Both in **open** and **closed** field lines,
(width ~20 Larmor radii)

In this region:
- Elevated magnetic shear \( s \)
- Elevated \( k_\theta \)

Both are stabilizing conditions for the interchange modes

(N. R. Walkden et al. NME 2016)
Spontaneous transport barrier in divertor geometry

**COMPASS-like DIVERTOR**

**LIMITER**

Enhanced pressure gradient in the closed flux surfaces, near the separatrix.
A narrow and stable transport barrier

Turbulent transport efficiency

\[ R_b = \frac{\left\langle \Gamma_E \right\rangle_{\theta, \phi}}{\left\langle \Gamma_{E} + \Gamma_{Diff} \right\rangle_{\theta, \phi}} \]

(E. Floriani et al. PPCF 2013)

- \( R_b = 0 \) strong barrier
- \( R_b = 1 \) no barrier

- Transport barrier with \( R_b \approx 0.3 \), varying in the range 0.2-0.6
- Stable character
- Small width (\( \sim 3 \rho_L \))
Damping of fluctuation amplitude

The level of fluctuations is reduced over the whole flux surface

Electric potential fluctuation amplitude reduced up to 50%

The level of fluctuations is reduced over the whole flux surface
Transport barrier acts mainly on low-amplitude fluctuations

Positive skewness \( S = \frac{\langle (N - \langle N \rangle_{t,\phi})^3 \rangle_{t,\phi}}{\langle (N - \langle N \rangle_{t,\phi})^2 \rangle_{t,\phi}^{3/2}} \)

→ Intermittent nature of turbulence

Low-amplitude events affected by the barrier

Effects of the barrier visible in \( \sim 15 \rho_L \) outside the separatrix
Two possible candidates for the barrier formation

Linearly stabilizing for interchange instabilities

Extremely strong in divertor in the vicinity of the X-point

BUT turbulence has a non-local character!

**magnetic shear**

\[ S_{cyl} = \frac{r}{q} \frac{dq}{dr} \]
Two possible candidates for the barrier formation

Moreover, a residual Reynolds stress can be driven by magnetic shear (see N. Fedorczak et al. *NF* 2012) → sinergy $E \times B$ and magnetic shear

\[ \partial_t <u_E^\theta>_\theta = - \frac{d}{dr} <\tilde{u}_E^\psi \tilde{u}_E^\theta>_\theta \]

Reynolds stress

Ongoing work suggests magnetic shear is the main player

In a circular geometry, an artificial magnetic shear is imposed with a divertor-like average q profile.

Transport barriers are triggered at the position where the magnetic shear is maximum. The $E \times B$ shear increases instead at the separatrix.

**Analogies**

- **Stability**
  - **Width** $\sim$ characteristic q decay length

**Differences**

- Lower amplitude
- Two peaks instead of one
Conclusions

- **TOKAM3X** simulations in diverted configuration show non-trivial effects of geometry on turbulent transport.

- **Turbulent structures** are almost field aligned, they have a **non-zero parallel wave-number** due to ballooning.

- Transverse fluxes are proportional to flux expansion. In order to understand turbulent transport, we have to remap to magnetic space.

- A significant turbulent transport is observed in the **outer divertor leg**, while turbulence is damped in the vicinity of the X-point.

- Unlike in limiter configuration, a spontaneous, stable **transport barrier** build-up near the separatrix, reducing turbulent transport by 70% at its centre.
Backslides
## Simulation parameters

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$\rho^* = \frac{\rho_L}{a} = \frac{1}{256}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics</td>
<td>$D_{N,\Gamma,W} = 2,5 \cdot 10^{-3}$ $\rho_L^2 \omega_{ci}$</td>
</tr>
<tr>
<td></td>
<td>$\eta_\parallel = 1,0 \cdot 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>$T_i = T_e = 1$</td>
</tr>
<tr>
<td>Particle source:</td>
<td>gaussian-shaped in the radial direction, centered at the inner boundary of the domain (edge) and poloidally constant</td>
</tr>
<tr>
<td>Numerics</td>
<td>$N_\theta = 512$ (SOL and core edge)</td>
</tr>
<tr>
<td></td>
<td>$N_r = 64$ (SOL + core edge)</td>
</tr>
<tr>
<td></td>
<td>$N_\varphi = 32$</td>
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</tbody>
</table>
Parallel flow trends are qualitatively recovered

Parallel Mach number is studied as global effect of cross-field transport

The main features of the parallel flows in the SOL are recovered

- Stagnation point between LFS midplane and LFS X-point
- Mach number close to 0.4 at the TOP of the machine
- Both small-scale turbulent flow and large-scale flows contribute to the build-up of this parallel flow scheme

\(N. \text{Asakura et al. JNM 2007}\)
Similar turbulence properties only far from X-point and separatrix

Density fluctuations PDF at LFS midplane

**LIMITER**

\[ r < a \quad \text{Skewness} \approx 0 \]

\[ r > a \quad \text{Skewness} > 0 \]

**DIVERTOR**

(P. Tamain CPP 2014)
Does flux expansion have a direct effect on turbulent transport?

Confinement improved by a low flux expansion at the LFS midplane

Different causes have been detected:
- Higher global curvature “g” in the inner shift case
- Stronger magnetic shear in the outer shift case can stabilize turbulence
Linear analysis of the system

(P. Tamain PhD thesis)
Residual Reynolds stress in divertor configuration

\[ \Pi \propto \langle \tilde{v}_r^2 \rangle _\theta < \frac{d v_E^\theta}{dr} \tau >_\theta - \langle \tilde{v}_r^2 \rangle _\theta < \hat{s} \theta F(\theta) >_\theta \]

(N. Fedorczak NF 2012)

Re stress depends on blobs shape

Top-bottom asymmetry introduced by the X-point causes a radial change of the average Reynolds stress

Source of poloidal flows
The vorticity equation contains information about the transport of poloidal momentum:

\[ \partial_t W + \nabla \cdot \left( W \frac{\Gamma}{N} \vec{b} \right) + \nabla \cdot (W \vec{u}_E) = \nabla \cdot (N \vec{u}_{vB}^i - N \vec{u}_{vB}^e) + \nabla \cdot (J \parallel \vec{b}) + D_W \nabla^2 W \]

Average on flux surface \( \langle . \rangle_{\theta, \phi} \), Closed field lines

\[ \partial_t < W > + \langle \nabla \cdot (W \vec{u}_E) \rangle = \langle \nabla \cdot (N \vec{u}_{vB}^i - N \vec{u}_{vB}^e) \rangle + \langle D_W \nabla^2 W \rangle \sim 0 \]

\( W \sim \frac{\partial^2 (\Phi + \ln(N))}{\partial^2 x} + \frac{\partial^2 (\Phi + \ln(N))}{\partial^2 y} \quad r \sim x \quad \theta \sim y \)

\[ < W >_y = \langle \frac{\partial^2 (\Phi + \ln(N))}{\partial^2 x} \rangle _y = \frac{d}{dx} < (v_{yEB}^y + v^*_y) >_y \]

\[ \partial_t \frac{d}{dx} < (v_{yEB}^y + v^*_y) >_y + \frac{d^2}{dx^2} < (\tilde{v}_{yEB}^y \tilde{v}_{EB}^x + \tilde{v}_y^* \tilde{v}_{EB}^x) >_y = D_W \frac{d^3}{dx^3} < (v_{yEB}^y + v^*_y) >_y \]

Reynolds stress

Diamagnetic Reynolds stress