The impact of anisotropy and flow on magnetic configuration and stability

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Expected impact of anisotropy

• Small angle $\theta_b$ between beam, field $\Rightarrow p_{||} > p_{\perp}$
• Beam orthogonal to field, $\theta_b=\pi/2 \Rightarrow p_{\perp} > p_{||}$
• If $p_{||}$ sig. enhanced by beam, $p_{||}$ surfaces distorted and displaced inward relative to flux surfaces
  [Cooper et al, Nuc. Fus. 20(8), 1980]
• If $p_{\perp} > p_{||}$, an increase will occur in centrifugal shift :
• Obtain $p_{\perp}$ and $p_{||}$ from moments of distribution function, computed by TRANSP
MHD with rotation & anisotropy

- Inclusion of anisotropy and flow in equilibrium MHD equations

\[ \nabla \cdot (\rho \mathbf{v}) = 0, \quad \rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{\bar{P}}, \quad \nabla \cdot \mathbf{B} = 0 \]

\[ \mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \]

\[ \mathbf{\bar{P}} = p_\perp \mathbf{I} + \Delta \mathbf{B} \mathbf{B}/\mu_0, \quad \Delta = \frac{\mu_0 (p_\parallel - p_\perp)}{B^2} \]

MHD with rotation & anisotropy


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- Frozen flux condition + axis-symmetry + neglect poloidal flow \( \Rightarrow \)

\[ \mathbf{v} = -R \phi'_E(\psi)e_\phi = R \Omega(\psi)e_\phi \quad \text{Equilibrium eqn becomes:} \]

\[ \nabla \cdot \left[ (1 - \Delta) \left( \frac{\nabla \psi}{R^2} \right) \right] = -\frac{\partial p_\parallel}{\partial \psi} - \rho H'(\psi) + \rho \frac{\partial W}{\partial \psi} - \frac{F(\psi)F'(\psi)}{R^2 (1 - \Delta)} + R^2 \rho \Omega(\psi)\Omega'(\psi) \]

\[ F = RB_\phi \quad H(\psi) = W_M(\rho, B, \psi) - \frac{1}{2} [R \phi'_E(\psi)]^2 \quad \text{Guiding-centre/MHD/double-adiabatic} \]
MHD with rotation & anisotropy

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- Frozen flux condition + axis-symmetry + neglect poloidal flow \( \Rightarrow \)

\[ \mathbf{v} = -R \phi'_E (\psi) \mathbf{e}_\varphi = R \Omega(\psi) \mathbf{e}_\varphi \]

Equilibrium eqn becomes:

\[ \nabla \cdot \left[ (1 - \Delta) \left( \frac{\nabla \psi}{R^2} \right) \right] = -\frac{\partial p_\parallel}{\partial \psi} - \rho H'(\psi) + \rho \frac{\partial W}{\partial \psi} - \frac{F(\psi) F'(\psi)}{R^2 (1 - \Delta)} + R^2 \rho \Omega(\psi) \Omega'(\psi) \]

\[ F = RB_\varphi \quad H(\psi) = W_M(\rho, B, \psi) - \frac{1}{2} [R \phi'_E(\psi)]^2 \]

Guiding-centre/MHD/double-adiabatic

- If two temperature Bi-Maxwellian model chosen

\[ p_\parallel (\rho, B \psi) = \frac{k_B}{m} \rho T_\parallel (\psi) \quad p_\perp (\rho, B \psi) = \frac{k_B}{m} \rho T_\perp (\psi) = \frac{k_B}{m} \rho T(\psi) \frac{B}{B - \theta(\psi) T_\parallel} \]

Set of 5 profile constraints \{F(\psi), \Omega(\psi), H(\psi), T_\parallel(\psi), \theta(\psi)\}
EFIT TENSOR: reconstruction code

M. Fitzgerald

- Adds kinetic constraints to magnetic-only constraints of EFIT
- Soloviev benchmarks computed for isotropic, anisotropic and flow
- Installed for both MAST and JET

J_\phi \text{ a strong function of transport model}

e.g. ITER-like plasma

\begin{table}
\begin{tabular}{|c|c|}
\hline
\epsilon & 0.4 \\
\sigma & 1 \\
\tau & 1 \\
R_0 & 6 m \\
B_0 & 5 T \\
\alpha & -3 \\
\rho_0 & 1 \times 10^{-7} \\
\Omega_0 & 0 or 7 \times 10^5 \text{ rad s}^{-1} \\
\Delta_0 & 0 or 4 \times 10^{-3} \\
I_p & 16 \text{ MA} \\
q^* & 1.6 \\
\beta_p & 1.0 \\
\beta_T & 0.07 \\
\hline
\end{tabular}
\end{table}

p_\perp/p_\parallel \sim 1.06

Extended Soloviev

\begin{align*}
p_\perp(R, B, \psi) &= \frac{1}{2} \rho_0 \Omega_0^2 R^2 - \frac{\Delta_0}{2} B^2 + \sigma_0 p_S(\psi) \\
p_\parallel(R, B, \psi) &= \frac{1}{2} \rho_0 \Omega_0^2 R^2 + \frac{\Delta_0}{2} B^2 + \sigma_0 p_S(\psi)
\end{align*}

\textbf{EFIT TENSOR}

\begin{align*}
p_\perp(\rho, B \psi) &= \frac{k_B}{m} \rho T_\perp(\psi) \\
p_\parallel(\rho, B \psi) &= \frac{k_B}{m} \rho T_\parallel(\psi)
\end{align*}
HELENA+ATF

• Companion code written to enable stability studies.
• Can be used to study how equilibrium changes with anisotropy

\[ J_\varphi = R \frac{B_p^2}{B^2} \left( \frac{\partial p_\parallel}{\partial \Psi} \right)_B + R \frac{B^2 - B_p^2}{B^2} \left( \frac{\partial p_\perp}{\partial \Psi} \right)_B \frac{1 - \Delta}{2R} \left( \frac{\partial (RB_\varphi)^2}{\partial \Psi} \right)_B \nabla \cdot \frac{\Delta \nabla \Psi}{R^2} \]
HELENA+ATF

- Companion code written to enable stability studies.
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\[ J_\phi = \frac{B_p^2}{B^2} \left( \frac{\partial p_\parallel}{\partial \Psi} \right)_B \left( \frac{\partial p_\perp}{\partial \Psi} \right)_B + \left( \frac{B^2 - B_p^2}{B^2} \right) \frac{1 - \Delta}{2R} \left( \frac{\partial (RB_\phi)^2}{\partial \Psi} \right)_B - R \nabla \cdot \frac{\Delta \nabla \Psi}{R^2} \]

MAST-like equilibrium

\[ J_\phi \text{ components} \]

\[ J_{\phi\text{nl}} \text{ core localized} \]

Anisotropy modifies poloidal current

- EFIT TENSOR reconstructions of MAST #18696 at 290ms
  - Anisotropic: $p_{\parallel}$ and $p_{\perp}$ constrained to values from TRANSP
  - Isotropic: $p^* = (p_{\parallel} + p_{\perp})/2$

\[ p_{\parallel}/p_{\perp} \approx 1.25 \text{ (anisotropic)} \]
\[ p^* = (p_{\parallel} + p_{\perp})/2 \text{ (isotropic)} \]

Most significant difference in $J_p$ which can effect change in stability

\[ \mu_0 J_p = \nabla (RB_\phi) \times \nabla \phi \]
Stability: New single adiabatic model

- Compressional
  - Double-adiabatic (CGL)
    - Collisionless, $p_\parallel$ and $p_\perp$ do independent work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit
  - New Single adiabatic (SA) model
    - $p_\parallel$ and $p_\perp$ doing joint work
    - Account for the isotropic part of the perturbation
    - Can reduce to MHD in isotropic limit
      - [Fitzgerald, Hole, Qu, PPCF 57 (2015) 025018]

- Incompressional
  - $p_{\parallel 1} = -\xi_n \left[ \frac{\partial p_\parallel}{\partial n} - (p_\parallel - p_\perp) \frac{\partial \ln B}{\partial n} \right]$  
    $p_{\perp 1} = -\xi_n \left[ \frac{\partial p_\perp}{\partial n} - (2p_\perp + \hat{c}) \frac{\partial \ln B}{\partial n} \right]$  

* A B Mikhailovskii, Instabilities in a confined plasma, IOP publishing (1998)

- Implemented in CSCAS (CSMIS-A) and MISHKA (MISHKA-A)
  - [Qu, Hole, Fitzgerald, PPCF 57 (2015) 095005]
MISHKA-A agrees with Bussac criterion

Z. Qu

- Benchmark result: Generalised Bussac condition: marginal stability of n=1 internal kink for $\varepsilon=0.1$, circular cross section

$$\frac{\langle p_{\perp} \rangle}{\langle p_{\parallel} \rangle} = \frac{1}{1 - \alpha(1 - \psi_n)}$$

Bussac condition = solid lines
MISHKA-A = points

[Qu, Hole, Fitzgerald, PPCF 57 (2015) 095005]
Adding anisotropy and flow to HAGIS

A. To handle anisotropy in equilibrium

**Formally**, change Hamiltonian field representation in HAGIS [e.g. W. A. Cooper *et al.* PPCF 53 (2011) 024001] & supply $J_{\phi, \text{aniso}}$

**or informally**, map $J_{\phi, \text{aniso}} \rightarrow \overline{J}_{\phi, \text{iso}}$ such that $\overline{q}_{\text{iso}} = q_{\text{aniso}}$, & supply $\overline{J}_{\phi, \text{iso}}$

If particle orbits for $\overline{J}_{\phi, \text{iso}}$ and $J_{\phi, \text{aniso}}$ are the same, no need to change Hamiltonian field representation in HAGIS

B. To handle anisotropy in mode structure

**Formally**, compute and supply $\delta B_{\text{aniso}}$ for $J_{\phi, \text{aniso}}$ [MISHKA-A]

**or informally**, compute and supply $\delta B_{\text{iso}}$ for $\overline{J}_{\phi, \text{iso}}$ [MISHKA]

For TAE’s, if $J_{\phi, \text{aniso}} \rightarrow \overline{J}_{\phi, \text{iso}}$ such that $\overline{q}_{\text{iso}} = q_{\text{aniso}}$

$\Rightarrow \delta B_{\text{aniso}} = \delta B_{\text{iso}}$

$\Rightarrow$ if *q* profile known exactly then anisotropy not relevant for spatial mode structure

C. Compute and supply $\omega$ [MISHKA-A]
Orbits for $\overline{J}_{\phi,\text{iso}}$ & $J_{\phi,\text{aniso}}$ are the same

Z. Qu, B. Layden, M. Fitzgerald

- Used CUEBIT to show impact of informal approach is small

\[ \overline{J}_{\phi,\text{iso}} \text{ such that } \overline{q}_{\text{iso}} = q_{\text{aniso}} \]
Adding anisotropy and flow to HAGIS

A. To handle anisotropy in equilibrium

Formally, change Hamiltonian field representation in HAGIS [e.g. W. A. Cooper et al PPCF 53 (2011) 024001] & supply $J_{\phi, \text{aniso}}$

or informally, map $J_{\phi, \text{aniso}} \rightarrow \overline{J}_{\phi, \text{iso}}$ such that $\overline{q}_{\text{iso}} = q_{\text{aniso}}$, & supply $\overline{J}_{\phi, \text{iso}}$

If particle orbits for $\overline{J}_{\phi, \text{iso}}$ and $J_{\phi, \text{aniso}}$ are the same, no need to change Hamiltonian field representation in HAGIS

B. To handle anisotropy in mode structure

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$\Rightarrow$ if $q$ profile known exactly then anisotropy not relevant for spatial mode structure

C. Compute and supply $\omega$ [MISHKA-A]
**Anisotropy on MAST: #29221**

- MAST #29221
- 1.6MW NB heating
- $I_p = 0.9\text{MA}, \beta_n \sim 3$
- Magnetics shows TAEs, tearing modes fishbones, long-lived modes

![Graph showing MAST shot #29221 parameters over time](image)
Beam + thermal population: \( \frac{p_\parallel}{p_\perp} \approx 1.7 \)

HELENA+ATF / EFIT TENSOR: \( p^* = \frac{p_\parallel + p_\perp}{2} \) (isotropic)

HELENA+ATF / EFIT TENSOR: \( p_\parallel, p_\perp \) (anisotropic)

\( p_\parallel/p_\perp = 1.7 \) at \( s=0.5 \) outboard
Beam + thermal population: $p_{\parallel} / p_{\perp} \approx 1.7$

**HELENA+ATF / EFIT TENSOR: $p^* = (p_{\parallel} + p_{\perp})/2$ (isotropic)**

![Graph showing isotropic tensor](image)

**HELENA+ATF / EFIT TENSOR: $p_{\parallel}, p_{\perp}$ (anisotropic)**

![Graph showing anisotropic tensor](image)

$p_{\parallel} / p_{\perp} = 1.7$ at $s=0.5$ outboard

- What is the impact on stability due to this q profile?
Incompressible continuum for MAST

\[ n = 1, \gamma = 0 \]

\[ R_{\text{mag}} = 0.914 \]
\[ f_A(R_{\text{mag}}) = 280 \text{kHz} \]

\[ R_{\text{mag}} = 0.928 \]
\[ f_A(R_{\text{mag}}) = 260 \text{kHz} \]

Isotropic \[ \Delta f_{\text{TAE}} \] < anisotropic \[ \Delta f_{\text{TAE}} \]

\[ \Rightarrow \text{anisotropic modes less susceptible to continuum damping} \]
Incompressible continuum for MAST

Z. Qu

MAST #29221 at 290ms.
n=1, γ=0

f_{A0} = 260kHz

Isotropic global mode
f = 81.3kHz

f_{A0} = 280kHz

Anisotropic global mode
f = 81.3kHz

Isotropic core mode
f = 83.5kHz

Isotropic global mode
f = 88.9kHz
Mode profile broader with anisotropy

Z. Qu

• Might saturation level be different for two cases?
Resonance maps from HAGIS

- Use linear eigenfunction from MISHKA+ATF to compute resonance maps ($\Omega=0$) of wave/particle evolution

$$\Omega = \omega - n\omega\phi - p\omega\theta$$

**isotropic,**

$n=1$, $p=1$,

$f = 88.9\text{kHz}$, $\Lambda = 0.3$

**anisotropic,**

$n=1$, $p=1$,

$f = 81.3\text{kHz}$, $\Lambda = 0.3$
Saturation level for (an)isotropic cases

B. Layden

• Distribution function is slowing down in energy, Gaussian in $s$ (centred at $s=0$) and $\Lambda$ (centred at $\Lambda = 0.3$).

• Preliminary results show anisotropic mode saturates at $(\delta B/B)_{\text{aniso}} = 0.1$ $(\delta B/B)_{\text{iso}} = 10^{-3}$.

Possible reasons include…. 

Anisotropic mode width broader

Resonances different

isotropic

anisotropic
Conclusions I

- Added anisotropy and toroidal flow to equilibrium reconstruction code EFIT TENSOR, and HELENA+ATF
- Developed new single adiabatic stability model which includes anisotropy and flow, reduces to ideal MHD as anisotropy and flow reduced
- Implemented Single Adiabatic CGL and incompressible stability treatments into continuum code CSMIS and stability code MISHKA-A
- Shown anisotropy (different q profile) changes the radial structure of TAE modes.
- Shown particle orbits largely unaffected if current profile remapped to have the same q profile.
Conclusions II

• Examined impact of anisotropy on a MAST case in detail:
  - q profiles of anisotropy / isotropic plasmas significantly different
  - Mode profile of anisotropic plasma broader than isotropic plasma
  - Resonance curves different
  - Saturation level of anisotropic plasma lower than isotropic plasma

• To do…
  - Compare predicted $\delta B/B$ to observed $\delta B/B$
  - Validate with fast particle diagnostics
  - Explore impact on lower frequency modes (e.g. BAE, BAAE, EGAM)
  - Explore the impact of anisotropy and flow on a wide range of MAST plasma conditions
  - Compute continuum damping for anisotropic plasmas (Builds on separate work in collaboration with Max Planck IPP to compute continuum damping in 3D)
Other ANU presentations

Poster Session I
• P15, *Pressure anisotropy and flow suppress diamagnetic holes in high-beta tokamaks*, B. Layden

Poster Session II
• P24, *Developments in advanced MHD Spectroscopy*, M.J. Hole, S. Cox, C. M. Ryu, M. H. Woo, K. Toi, J. Bak, S. Sharapov, M. Fitzgerald
• P26, *Flow enabled instabilities in energetic geodesic acoustic modes (EGAMs)*, Z. Qu
Auxiliary slides
“MHD with anisotropy in velocity, pressure”

• Pressure different parallel and perpendicular to field due mainly to directed neutral beam injection

⇒ Pressure is a tensor \[ \overline{P} = p_\perp \overline{I} + \Delta \overline{BB} / \mu_0, \]
\[ \Delta = \frac{\mu_0 (p_\parallel - p_\perp)}{B^2} \]