Comparing MHD simulations to experiments

K. J. McCollam (kmccollam@wisc.edu), A. F. Almagri, C. M. Jacobson, J. A. Reusch, J. S. Sarff, J. P. Sauppe, C. R. Sovinec, J. C. Triana, and V. V. Mirnov, University of Wisconsin

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Sawtooth relaxation in the reversed-field pinch (RFP) provides test case for validating extended MHD

- Visco-resistive MHD can be augmented with two fluids, finite beta, kinetic effects, impurity evolution, radiation, ... 
- Validation is more robust using a larger configuration space
Outline

- Comments on validation
- RFPs and sawtooth relaxation
- Single-fluid MHD
- Two-fluid MHD with gyroviscosity
Comments on validation
Rigorous validation for accurate prediction of extrapolated burning plasma cases will be expensive.
Formal validation requires uncertainty quantification for nonlinear simulations as well as experiments

- A single run for fusion-relevant parameters is expensive

- Only established uncertainty quantification method is sampling multidimensional inputs for multiple runs

Formal validation for fusion still in early stages

- Nonlinear simulation uncertainties often not calculated
- Physics understanding and predictive capability require more than validation alone
  - Experimental exploration
  - Interpretive simulations
  - Analytic theory
- Example from MHD: how to numerically treat fluid viscosity in the presence of stochastic magnetic fields?
RFPs and sawtooth relaxation
Reversed-field pinch (RFP) extends configuration space outside tokamaks

- High beta
- Highly sheared magnetic field
- Multiple tearing-mode resonances allow complex nonlinear MHD
Madison Symmetric Torus (MST)

- $R_0/a = (1.5 \text{ m})/(0.52 \text{ m})$
- Aluminum shell 5 cm thick
- $I_p \lesssim 600 \text{ kA}$
- $T_{e,i} \lesssim 2 \text{ kV}$
- $n_e \sim 10^{19}/\text{m}^3$
- Lundquist number $S \equiv \tau_{\text{res}}/\tau_A \lesssim 10^7$
- Advanced diagnostics for profile measurements of magnetic field, density, temperature, flow, ...
Sawtooth relaxation is a limit-cycle phenomenon in MST

- Ohmic drives $\lambda \propto J_\parallel / B$
  $\propto 1 - \rho^\alpha$ more peaked:
  flatness parameter $\alpha$ decreases

- Core-resonant $m = 1$ modes become unstable

- Edge-resonant $m = 0$ stable but nonlinearly driven by $m = 1$ at sawtooth crash

- Crash EMF generates core toroidal flux $\Phi$, flattens $\lambda$

- Key physics mechanism: fluctuation-induced EMF in mean-field parallel Ohm’s law,
  $\langle E \rangle_\parallel \simeq -\langle \tilde{V} \times \tilde{B} \rangle_\parallel + \langle \tilde{J} \times \tilde{B} \rangle_\parallel / (en) + \langle \eta J \rangle_\parallel$
Single-fluid MHD
Single-fluid MHD in cylindrical geometry with the DEBS code

- \[ \frac{\partial \mathbf{A}}{\partial t} = \mathbf{S} \mathbf{V} \times \mathbf{B} - \eta \mathbf{J} \]
- \[ \rho \frac{\partial \mathbf{V}}{\partial t} = -\mathbf{S} \rho \mathbf{V} \cdot \nabla \mathbf{V} + \mathbf{S} \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V} \]
- This run:
  - \( \beta = 0 \)
  - Lundquist number \( S = 3.8 \times 10^6 \) matches MST experiments
  - Magnetic Prandtl number \( P_m = \text{‘viscosity/resistivity’} \sim 100 \)
  - Viscosity unphysically adjusted in time according to magnitude of perturbed flows to damp subgrid-scale fluctuations
  - No external circuit model or pressure profile effects
  - J. Reusch et al., PRL 107, 155002 (2011)
MST sawtooth cycles qualitatively reproduced
Equilibrium evolution agrees well
Local metrics show where improvements in agreement needed

\[ \chi^2(r, t) = \left( \frac{y_{\text{sim}}(r, t) - y_{\text{exp}}(r, t)}{\sigma_{\text{sim}}(r, t) + \sigma_{\text{exp}}(r, t)} \right)^2 \]
MST magnetic fluctuation amplitudes strongly overpredicted

- Long-term MST goal is an extended MHD validation study of the $S$ scaling of fluctuation amplitudes
Two-fluid MHD with gyroviscosity
Extended MHD with the NIMROD code


\[ E = -\mathbf{V} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{en} - \frac{\nabla p_e}{en} + \eta \mathbf{J} + \frac{m_e}{e^2 n} \frac{\partial \mathbf{J}}{\partial t} \]

\[ \rho_i \frac{d \mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_{gv} - \nabla \cdot \nu \rho_i \mathbf{W} \]

- Hall term \( \mathbf{J} \times \mathbf{B} / (en) \)

- Ion gyroviscous stress \( \Pi_{gv} \)

- These runs:
  - Cylindrical geometry
  - Single fluid or two-fluid with cold or warm (\( \beta = 0.1 \)) ions
  - \( S \leq 8 \times 10^4 \), much smaller than most MST cases
  - \( P_m \leq 1 \), similar to MST perpendicular value
  - No external circuit model or pressure profile effects
Two-fluid MHD with ion gyroviscosity has saturated magnetic-fluctuation amplitudes 2x smaller than single-fluid

• Tends to agree better with MST experiments

• King, Sovinec, & Mirnov, POP 19, 055905 (2012)
Hall term due to fluctuations has complex radial structure in both simulation and experiment

Simulation

Experiment

\[ \frac{\dot{E}}{\nu_A B_0} \]

\[ \eta J || \]

\[ -\langle \dot{v} \times \dot{B} \rangle || \]

\[ \langle \dot{J} \times \dot{B} / ne \rangle || \]

King, Sovinec, and Mirnov, POP 19, 055905 (2012)

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- We are only now approaching formal validation of extended MHD
Two-fluid MHD relaxation events lead to changes in mean flow profile, as in MST experiments

- Flow profile in second event is flattened (due to Hall term in red) matching experiment
- Note single-fluid MHD relaxation does not change the flow profile
Summary

- Comments on validation
  - Fusion validation an expensive project only just begun

- RFP and sawtooth relaxation
  - Test case for comparing extended MHD models to experiment

- Single-fluid MHD
  - These DEBS runs qualitatively reproduce MST equilibrium evolution but overpredict magnetic fluctuation amplitudes

- Two-fluid MHD with gyroviscosity
  - These NIMROD runs better match MST magnetic fluctuation amplitudes and flow profile evolution