Use of Reliability Assessment Method to Quantify Probabilistic Safety of Reactor Thermal Hydraulic Parameters

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What is Probabilistic Safety Analysis (PSA)?

Safety Analysis of a System based on probabilistic concept. This term does not imply any particular approach or any particular interpretation of probability.

Why PSA is necessary?

Safety margin inherent in the design for operation alter due to uncertainties arise from the factors such as crack like defects, degradation of material strength, accuracy and frequency of pre- and in service inspections and service loadings etc. It means it is not realistic in most problems to assign specific, constants values to this parameters. These parameters can be best represented statistical distributions rather than single values representing absolute certainty.
Levels of PSA for Reactor Safety Analysis

Level 1 PSA (Core Damage frequency)

Level 2 PSA (Containment Failure Frequency with Source Term Analysis)

Level 3 PSA (Risk Analysis)
Reliability is the probabilistic measure of structural safety and it is the numerical value obtained by the reliability analysis. Staring from the principles of limit state analysis, reliability analysis is then applied to the codified design to calculate the probability of failure or unacceptable performance of the structural component.
How reliability analysis can help?

Reliability analysis enables engineers to tackle the following:

- Analysis
- Quantification
- To understand the requirement for frequency of inspection
- To perform risk assessment
- To take appropriate decision whether the structure is at the design stage or in the construction stage or in actual use.
Uncertainties in Reliability Analysis

The accuracy in reliability measurement depends on the analyst’s present state of knowledge and experience about the structure, and the computed reliability is likely to change as more information becomes available. Lack of information about the uncertainties involved with different parameters may result in the prediction of structural reliability being inaccurate. These uncertainties can arise from a large number of sources:

- Physical Uncertainties
- Statistical Uncertainties
- Model Uncertainties
- Human Errors
The structural performance of a complete structure or part of it is generally described by a specified set of limit states that separate the desired states of the structure from adverse states. In a reliability context, the random variables associated with each limit state need to be identified. Therefore, it is necessary to group these variables into demand and capacity variables so that a safety margin equation can be defined. Then the system failure is given by single limit state function, say

\[ g(R, S) = R - S \leq 0 \]
where *S* is the *load* variable known as demand variable, and *R* is the resistance effect variable known as a capacity variable.

More generally, if there are *n* basic random variables that represent a vector of *n* basic variables, the safety margin or limit state equation can then be formulated as

\[
M = g(\bar{X}) = g(X_1, X_2, \ldots, X_n)
\]

Above equation is the safety margin equation, and the reliability or the probability of non-violation of the limit state can be expressed as

\[
R = 1 - P_f = 1 - P [M \leq 0]
\]
Method for Estimating Reliability
Reliability Assessment Methods

- First Order Reliability Method (FORM)
- Second Order Reliability Methods (SORM)
- Monte Carlo Simulation Method

Variance Reduction Techniques

- Importance Sampling Method
- Directional Simulation Method
**FORM/SORM Method**

\[ \beta_c = \frac{\mu_M}{\sigma_M} \]

\[ R = 1 - P_f = 1 - \Phi(-\beta_c) \]
Limitations of FORM/SORM
Limitations of FORM/SORM
If $\bar{x}$ represents a random vector in $n$-dimensional space and $g(\bar{x})$ is the safety margin function of $\bar{x}$, then the probability of failure of the structure according to classical reliability theory is

$$P_f = \int_{g(\bar{x}) \leq 0} f_{\bar{x}}(\bar{x}) \, d\bar{x}$$

where $f_{\bar{x}}(\bar{x})$ is the joint probability density function for the independent variables $\bar{x}$ and where the integral is over the failure domain $g(\bar{x}) \leq 0$. 
Having transformed the physical variables $\bar{X}$ in $Z$-space, using the indicator function $I\{\cdot\}$

$$P_f = \int_{\bar{Z}} I\{g(\bar{Z}) \leq 0\} f_{\bar{Z}}(\bar{Z}) \, d\bar{Z}$$

with $I\{g(\bar{Z})\} = 1$ for $g(\bar{Z}) \leq 0$ i.e. failure, and

$I\{g(\bar{Z})\} = 0$ for $g(\bar{Z}) > 0$ i.e. not failure.
Crude Monte Carlo Simulation

From the definition of expectation, the above expression in relation to the number of random trials $q$ will be

$$P_f = E[I(g(\bar{Z}))] = \frac{1}{q} \sum_{j=1}^{q} I[g(\bar{Z}_j)]$$

which approaches the exact failure probability $P_f$ when $q$ approaches infinity.
Crude Monte Carlo Simulation

*Failure region*

\[ g(z_R, z_S) = 0 \]

\[ \beta = 2.0 \]

10,000 trials
The estimate of failure probability $P_f$ is itself a random variable with its own mean and variance. In crude Monte Carlo analysis, this variance is dependent only on the number of trials and the true value of $P_f$. A number of so called variance reduction techniques are available which serve to improve the efficiency of the calculation considerably by reducing the number of trials so that the solution will be obtained within a small confidence range. Two such techniques which are useful in reliability studies are importance sampling and directional simulation. For these methods to be used a certain amount of additional information is required- for example, the location of the region of the sampling space which is going to contribute most to the failure probability. Only directional simulation is discussed here.
Application of the Methods

The methods that we shall be examining originated in the field of structural engineering where high levels of reliability are normally to be expected such as in advanced systems: aerospace structures, offshore oil and gas production, nuclear pressure vessels or system, etc. However, these methods have wide applicability now a days in other fields such as electrical, mechanical, electronic even in financial systems, although in some areas the full potentials is still to be realized.

Application in Fracture Mechanics

Probabilistic Fracture Mechanics (PFM), which blends fracture mechanics with the theory of probability and statistics, advances the reliability assessment of structural systems. It plays an important role in probabilistic safety assessment of systems with crack-like defects. PFM focuses on the sizes and shapes of cracks, their location and orientation and their probabilistic descriptions.
Developing a reliability-based methodology for the fracture integrity assessment of a structural component using Finite Element Method (FEM)

Research Tool: ABAQUS and ZENCRACK

Fracture failure of an expansion loop containing a crack

- Typical geometry
- Ductile Material
FE Modelling and Analysis
Initiation Based Fracture Failure

\[ g(Z_P, Z_T, Z_Y, Z_{JIC}) = J_{IC}(Z_{JIC}) - J(Z_P, Z_T, Z_Y) \leq 0 \]

- \( Z_P \sim \) Pressure, \( P \), Gumbel Variable
- \( Z_T \sim \) Temperature, \( T \), Normal Variable
- \( Z_Y \sim \) Material Yielding, \( Y \), Lognormal Variable + Temperature Dependency
- \( Z_{JIC} \sim \) Fracture Toughness, \( J_{IC} \), Lognormal Variable
Directional Simulation Method

\[ P_f = \frac{1}{q} \sum_{j=1}^{q} \left( 1 - \chi_2^2 \{ r_f^2 \} \right) \]

\( q \sim \text{number of trials}, \ r_f \text{ is the failure surface distance from the origin} \)

\( M = g(Z) = 0 \)
The present problem is in four-dimensional Space:

\[ g(Z_P, Z_T, Z_Y, Z_{JIC}) = J_{IC}(Z_{JIC}) - J(Z_P, Z_T, Z_Y) \leq 0 \]

\[ P_f = \frac{1}{q} \sum_{j=1}^{q} \left( 1 - \chi^2_{4} \{ r_j^2 \} \right) \]
Box and Muller Method for Random Number Generation

It produces a pair of independent standard normal variates given by

\[
\begin{align*}
z_1 &= \left(-2 \ln u_1\right)^{1/2} \cos(2\pi u_2) \\
z_2 &= \left(-2 \ln u_1\right)^{1/2} \sin(2\pi u_2)
\end{align*}
\]

where \( u_1 \) and \( u_2 \) are independent random variables from the same rectangular distribution in the interval \([0,1]\).
Calculation of $r_f$ for $q=75$ trials

$$P_f = \frac{1}{q} \sum_{j=1}^{q} \left( 1 - \chi_4^2 \{ r_j^2 \} \right)$$

$SM = J_{IC} - J \geq 0$
Probability of failure with respect to the crack depth
## Probability of failure with respect to the crack depth

### Results

<table>
<thead>
<tr>
<th>Crack Length</th>
<th>Crack Depth (mm)</th>
<th>$P_f$</th>
<th>$\beta$</th>
<th>Smallest $r_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70:21</td>
<td>70:21</td>
<td>8.556E-2</td>
<td>1.368</td>
<td>1.6818</td>
</tr>
<tr>
<td>70:20</td>
<td>70:20</td>
<td>1.889E-2</td>
<td>2.077</td>
<td>2.4181</td>
</tr>
<tr>
<td>70:19</td>
<td>70:19</td>
<td>4.462E-3</td>
<td>2.614</td>
<td>2.7636</td>
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<tr>
<td>70:18</td>
<td>70:18</td>
<td>1.887E-3</td>
<td>2.896</td>
<td>3.000</td>
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<tr>
<td>70:15</td>
<td>70:15</td>
<td>4.5844E-4</td>
<td>3.314</td>
<td>3.3660</td>
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<tr>
<td>70:12</td>
<td>70:12</td>
<td>8.5710E-5</td>
<td>3.758</td>
<td>3.8863</td>
</tr>
<tr>
<td>70:10</td>
<td>70:10</td>
<td>4.3862E-5</td>
<td>3.928</td>
<td>4.0880</td>
</tr>
</tbody>
</table>
Application in Reactor Core Thermal Hydraulics

Research Tool: COOLOD-N2 Code

Core arrangement of the TRIGA reactor.

Stainless Steel
Top End Fitting

Graphite

Uranium
Zirconium
Hydride

Graphite

Stainless Steel
Bottom End Fitting

TRIGA LEU fuel
Heat Source
Two major design criteria were set up for the thermal hydraulic design so that fuel rods may have enough safety margins for the condition of normal operation.

(1) To avoid nucleate boiling of coolant anywhere in the core in order to give enough allowance against the burnout of the fuel elements even at the hottest spot in the core.

(2) To give enough margin against the burnout itself of the fuel elements under the condition of normal operation so that there may be enough margin also for operational transients. The departure from the nucleate boiling ratio (DNBR) was decided to be not less than 1.5 to meet this criterion.
Subject of Thermal Hydraulics

- Power distribution in the reactor fuel cells
- Heat conduction in fuel rods
- Heat Transportation from fuel to coolant flow
- Flow distribution and heat balance
- Peak fuel temperature
- Onset of nucleate boiling
- Critical heat flux/DNB heat flux
- Flow instability
- Vibrations and deformation of fuel plates
Thermal-hydraulic Analysis Procedures

- Power distribution
- Pressure loss
- Flow rate

Heat Transportation

- Flow distribution
- Coolant temperature

ONB conditions

- DNB heat flux

Heat transfer coefficient

Fuel surface temperature

Fuel clad temperature
Steady State Thermal-Hydraulic Analysis Codes

- PLTEMP/ANL.................ANL
- NATCON........................ANL
- MATRA-h......................HANARO
- COOLOD-N2...................JAEA
- HEATHYD.....................IAEA
- STAT...........................TRIGA
- NCTRIGA......................TRIGA

Transient Thermal-Hydraulic Analysis Codes

- EUREKA2/RR..................JAEA
- PARET..........................ANL
- RELAP5........................NRC
- COBRA..........................IAEA
- MARS............................HANARO
The research reactor TRIGA Mark-II is a 3MW pool type, light water moderated and cooled reactor with 19.7 % LEU Rod type fuels.

Steady State Condition or Normal Operational Condition. Two modes are adapted for core cooling, these are:

- One is a natural circulation cooling mode for a "Low power range" of up to 500KW
- The other is a forced-convection cooling mode for a "High power range", of up to 3MW.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Design Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Element</td>
<td>20% w/o U-ZrH, 19.7% enriched</td>
</tr>
<tr>
<td>Total number of fuel in the core</td>
<td>100</td>
</tr>
<tr>
<td>Cladding</td>
<td>Stainless Steel 304L</td>
</tr>
<tr>
<td>Reflector</td>
<td>Graphite</td>
</tr>
<tr>
<td>Inlet Temperature °C</td>
<td>40.6</td>
</tr>
<tr>
<td>Radius of Zr rod (cm)</td>
<td>0.3175</td>
</tr>
</tbody>
</table>
# Outline of TRIGA Mark-II Research Reactor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel radius (cm)</td>
<td>1.82245</td>
<td>Fuel radius (cm)</td>
<td>1.87706</td>
</tr>
<tr>
<td>Clad outer radius (cm)</td>
<td>1.87706</td>
<td>Clad outer radius (cm)</td>
<td></td>
</tr>
<tr>
<td>Gap width (cm)</td>
<td>0.00381</td>
<td>Gap width (cm)</td>
<td></td>
</tr>
<tr>
<td>Active fuel length (cm)</td>
<td>38.1</td>
<td>Active fuel length (cm)</td>
<td></td>
</tr>
<tr>
<td>Flow area (cm²)</td>
<td>5.3326</td>
<td>Flow area (cm²)</td>
<td></td>
</tr>
<tr>
<td>Hydraulic Diameter (cm)</td>
<td>1.80594</td>
<td>Hydraulic Diameter (cm)</td>
<td></td>
</tr>
<tr>
<td>Pressure (Pa)</td>
<td>$1.60654 \times 10^5$</td>
<td>Pressure (Pa)</td>
<td></td>
</tr>
</tbody>
</table>
## Outline of TRIGA Mark-II Research Reactor

<table>
<thead>
<tr>
<th>Friction Loss Coefficient</th>
<th>0.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Loss Coefficient</td>
<td>1.81 (inlet); 2.12 (Outlet)</td>
</tr>
<tr>
<td>Pitch (cm)</td>
<td>4.5716</td>
</tr>
<tr>
<td><strong>Coolant Velocity (cm/sec)</strong></td>
<td></td>
</tr>
<tr>
<td>(a) Natural Convection Mode</td>
<td>30.48</td>
</tr>
<tr>
<td>(b) Forced Flow</td>
<td>287.58</td>
</tr>
<tr>
<td><strong>Mass Flow rate, kg/m²s</strong></td>
<td></td>
</tr>
<tr>
<td>(a) Natural Convection Mode</td>
<td>145.20</td>
</tr>
<tr>
<td>(b) Forced Convection Mode</td>
<td>$3.2089 \times 10^3$</td>
</tr>
</tbody>
</table>
Fuel Rod Temperature of TRIGA Type Reactor

![Diagram of fuel rod structure with labels for CL (cladding), coolant, clad, gap, heat conduction, fuel meat, and fuel rod.]
Heat Conduction in a Fuel Rod

Transient 1D heat conduction equation

\[ c \rho \frac{\partial T}{\partial t} = \frac{k}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + Q \]

- \( Q \): heat generation density
- \( c \): specific heat
- \( k \): thermal conductivity

Steady \( \Rightarrow \)
\[ \frac{k}{r} \frac{d}{dr} \left( r \frac{dT_f}{dr} \right) + Q = 0 \]

\( k_f \), \( Q \) : constant

\[ \frac{dT_f}{dr} = 0, \quad T_f = T_m \quad \text{at} \quad r = 0 \]

\[ T_f(r) = T_m - \frac{Q}{4k_f} r^2 \]

\[ T_m - T_s = \frac{QR_s^2}{4k_f} = \frac{q_s R_s}{2k_f} \]

\[ q_s = \frac{QR_s}{2} \]

Clad
\[ Q = 0 \]

\[ \frac{k_c}{r} \frac{d}{dr} \left( r \frac{dT_c}{dr} \right) = 0 \]

\[ T_c(r) = \frac{q_s R_s}{k_c} \ln \frac{R_g}{r} + T_g, \quad T_g - T_w = \frac{q_s R_s}{k_c} \ln \frac{R_w}{R_g}, \quad T_m - T_w = q_s \left( \frac{R_s}{2k_f} + \frac{R_w - R_s}{k_g} + \frac{R_s}{k_c} \ln \frac{R_w}{R_g} \right) \]
**Gap Conductance**

- **uniform gap**

  \[ q_G = h_g(T_s - T_g) \]

  \[ h_{g,\text{open}} = \frac{k_{\text{gas}}}{\delta_{\text{eff}}} + \frac{\sigma}{\varepsilon_f} \left( \frac{1}{\varepsilon_f} + \frac{1}{\varepsilon_c} - 1 \right) \frac{T_s^4 - T_g^4}{T_s - T_g} \]

  - \( \delta_{\text{eff}} \): effective gap width \((R_g - R_s)\)
  - \( k_{\text{gas}} \): thermal conductivity of gas

- **effect of solid conductance**

  \[ h_g = h_{g,\text{open}} + h_{\text{contact}} \]

  **(theoretical)**

  \[ h_{\text{contact}} = C \frac{2k_f k_c}{k_f + k_c} \cdot \frac{p_i}{H \sqrt{\delta_{\text{eff}}}} \]

  - \( p_i \): contact pressure
  - \( H \): Meyer’s hardness number
  - \( C \): constant
Fluid Equations

- **Fluid mass equation**

\[ A \frac{\partial \rho}{\partial t} = - \frac{\partial w}{\partial x} \]

- **Fluid momentum equation**

\[ A \frac{\partial (\rho v)}{\partial t} = - \frac{\partial (w v)}{\partial x} - A \frac{\partial p}{\partial x} - \rho g A \frac{\partial z}{\partial x} - \frac{\partial F_k}{\partial x} \]

- **Fluid energy equation**

\[ A \frac{\partial (\rho e)}{\partial t} = - \frac{\partial}{\partial x} \left[ \rho \left( h + \frac{v^2}{2} + \Phi \right) \right] + q_w \frac{\partial A_w}{\partial x} \]

\[ e = u + \frac{v^2}{2} + \Phi \]

\[ h = u + \frac{p}{\rho} \]

\[ g = \frac{\partial \Phi}{\partial z} \]

**Symbols:**
- \( \rho \): fluid density
- \( v \): fluid velocity
- \( p \): thermodynamic pressure
- \( F_k \): frictional force
- \( e \): fluid specific energy
- \( u \): fluid specific internal energy
- \( h \): fluid enthalpy
- \( \Phi \): gravity potential function
- \( A_w \): wall heated area
- \( t \): time
**Definition of DNBR**

\[
Minimum\,DNBR = \frac{q_{CHF}}{q_{max}}
\]

**Safety Margin Equation**

\[
g(Z_{min}, Z_T) = DNBR(Z_{min}) - DNBR(Z_T) \leq 0
\]

- \(Z_T\) ~ Inlet Temperature, Normal Variable
- \(DNBR(Z_{min})\) ~ Minimum DNBR, Normal Variable
Probability of failure with respect to Rod Hot Factor for 50 trials
Conclusion

It is seen that the probability of failure increases with increase in rod factor values, which is expected. Hence, it seems the method works well in determining fuel failure probability. However, this is indeed a very preliminary attempt where reactor steady state has been considered. As a part of reactor safety analysis, the authors at present have given focus to calculate the fuel failure probability as a function induced reactivity considering reactivity initiated transient analysis. It can be concluded that the method opens a new area for further research in reactor thermal hydraulics.
References


Thank You.