Temperature Analysis and Failure Probability of the Fuel Element in HTR-PM

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Abstract — Spherical fuel element is applied in the 200-MW High Temperature Reactor-Pebble-bed Modular (HTR-PM). Each spherical fuel element contains approximately 12,000 coated fuel particles in the inner graphite matrix with a diameter of 50mm to form the fuel zone, while the outer shell with a thickness of 5mm is a fuel-free zone made up of the same graphite material. Under high burnup irradiation, the temperature of fuel element rises and the stress will result in the damage of fuel element. The purpose of this study is to analyze the temperature of fuel element and to discuss the stress and failure probability.

Key words- HTR-PM, fuel element, temperature analysis, failure probability

I. INTRODUCTION

High temperature gas-cooled reactor belongs to the 4th generation reactor. It features with inherent safety, high efficiency, capable of hydrogen production[1][2]. High Temperature Reactor-Pebble-bed Modular (HTR-PM), which consisted of two 250 MWth reactor-steam generator modules and one steam turbine-generator set, is being built in Shidao Bay, Weihai City, China[3].

In HTR-PM, spherical fuel elements are placed on the pebble bed. The helium is swept around the pebbles to cool them down and to evacuate the heat.

In HTR-PM, spherical fuel elements with TRISO coated particles are used, which have proven capability of fission product retention up to 1600°C in case of accident. The TRISO coated particles has four layers outside the fuel kernel. The four layers are composed by porous carbon buffer layer, inner pyrolytic carbon layer, silicon carbide barrier coating layer, and outer pyrolytic carbon layer. The spherical fuel elements with 60mm in diameter are composed of a fuel zone with diameter of 50mm and a fuel-free shell with thickness of 5mm. In the fuel zone, 12000 TRISO coated particles are evenly distributed in the graphite matrix[4-6].

Graphite matrix, served as a structural material in the fuel element, needs to execute various tasks as follows. Firstly, it should have high thermal conductivity, so that it could transfer fission heat from coated fuel particles to the outer surface of the fuel elements. Secondly, it should have high mechanical strength, so as to endure various external forces. It is critical to reduce external forces because fuel elements have sustained different kinds of external forces in the process of transporting, loading and unloading. For instance, during the process of loading into the reactor core, spherical fuel elements will fall down on the pebble bed. In addition, graphite matrix also should have high density, good corrosion resistance and good irradiation performance[7].

Under high burnup irradiation, the temperature of fuel element rises and the stress due to high-temperature irradiation will cause fuel element broken. In this paper, two models are built, and the temperature of the fuel element is analyzed, besides, the stress and failure probability are researched.

II. BUILD UP THE MODEL

Taking the structure of the spherical fuel element into consideration, the model is built up with two parts which are fuel zone and fuel-free shell. The model of the spherical fuel element is shown in Fig 1. In the process of reactor operation, plenty of heat...
are generated in the fuel area since nuclear fission occurred in the TRISO coated particles. Graphite matrix transfers the heat from TRISO coated particles to the outer surface of the fuel elements. And in the process, the graphite matrix shrinks and deforms because of the heat and irradiation. When the deformation force exceeds the permitted stress of the graphite matrix, the spherical fuel elements will be broken. The model of spherical fuel element is built up to analyze the distribution of temperature, stress and failure probability. Hence, the model consists of two parts: which are temperature analysis model and failure probability analysis model.

II.A. Temperature Analysis Model

In regard to the temperature analysis model, two specific and different models are applied. In model1, fuel zone is regarded as an organic whole. That is, fuel zone is considered as one internal heat source. The fuel-free shell is then considered as a shell containing internal heat source.

According to the heat conduction equation, under spherical coordinates, temperature varies in according to the change of radius. The influence of time is excluded. Then,

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \Phi = 0 \quad \text{(eq. 1)}
\]

Where, \( \Phi \) is power density, \( r \) is radius, and \( T \) is temperature.

The third boundary condition is shown as below,

\[
- \lambda \frac{\partial T}{\partial r} \bigg|_w = H(T_w - T_g) \quad \text{(eq. 2)}
\]

Where, \( H \) is the heat emission coefficient of graphite, \( T_w \) is the temperature on the point \( w \), \( T_g \) is the temperature of helium.

To put the third boundary condition into the equation 1, the following equations are obtained.

In the fuel zone, the temperature distribution is shown in equation 3.

\[
T(r) = \frac{q_1(a^2 - r^2)}{6\lambda} + \frac{q_2(b^2 - r^2)}{6\lambda} + \frac{(q_1 + q_2) a^3}{3\lambda} \left( \frac{1}{a} - \frac{1}{b} \right) + \frac{(q_1 - q_2) a^3}{3b^2H} + \frac{q_1b}{3H} + T_g \quad \text{(eq. 3)}
\]

In the fuel-free shell, the temperature distribution is shown in equation 4.

\[
T(r) = \frac{q_1(b^2 - r^2)}{6\lambda} + \frac{(q_1 + q_2) a^3}{3\lambda} \left( \frac{1}{r} - \frac{1}{b} \right) + \frac{(q_1 - q_2) a^3}{3b^2H} + \frac{q_1b}{3H} + T_g \quad \text{(eq. 4)}
\]

Where, \( a \) is the radius of fuel zone, \( b \) is the radius of fuel-free shell, \( q_1 \) is the power density in the fuel zone, \( q_2 \) is the power density in the fuel-free shell.

For every spherical fuel element, it is assumed that, fission power comes from the kinetic energy of fission fragment is 80%, which is well-distributed in the fuel zone, and heating power comes from the \( \gamma \) ray is 20%, which is well-distributed in the whole fuel sphere.

\[
q_1 = \frac{0.8q}{3} ab^3 \quad \text{(eq. 5)}
\]

\[
q_2 = \frac{0.2q}{3} ab^3 \quad \text{(eq. 6)}
\]

Where, \( q \) is power density of the fuel sphere.

In model1, fuel zone is supposed as an organic whole heat source. However, plenty of small heat source are diffused in the fuel zone. In this case, each TRISO coated particle is a heat source. Therefore, the assumption of the model 1 is conservative[8]. To consider the influence of mass heat source, model 2 is built up.

In model 2, fuel zone is equally divided into 12000 parts. Every part is simplified into a sphere with internal heat source. The heat source is TRISO coated particle. The radius of the equivalent sphere is calculated by its volume. For every equivalent sphere, the temperature distribution is calculated by equation 3. It is important to note that, \( a \) is instead of the radius of TRISO coated particle, and \( b \) is instead of the radius of equivalent sphere. For the fuel-free shell, the temperature distribution is still calculated by equation 4.

The schematic diagram of equivalent sphere is shown in Fig 2.

Fig. 1: The model of the spherical fuel element.
II.B. Failure Probability Analysis Model

In the spherical fuel element, the volume of TRISO coated particles is extremely limited, even less than 6%. Consequently, in the stress analysis model, the spherical fuel element is considered as a graphite solid sphere. Meanwhile, the TRISO coated particles are ignored and the graphite material is assumed isotropic. In accordance with the linear viscoelastic theory, under spherical coordinates, the stress and strain equations are as follows:[9]

\[
\begin{align*}
\sigma_r(D) & = \frac{1}{(1 + \mu)(1 - 2\mu)} \left[ \int_0^D G(D - D') \left( \frac{\partial \varepsilon_{\theta}}{\partial D} \right)_r \right] dD' + 2\mu \int_0^D G(D - D') \frac{\partial \varepsilon_\theta(D')}{\partial D} dD' \\
\sigma_\theta(D) & = \sigma_\phi(D) = \frac{\mu}{(1 + \mu)(1 - 2\mu)} \left[ \int_0^D G(D - D') \left( \frac{\partial \varepsilon_\theta}{\partial D} \right)_r \right] dD' + \int_0^D G(D - D') \left( \frac{\partial \varepsilon_\theta}{\partial D} \right)_r dD'
\end{align*}
\]

(eq. 7)

Where, \( \mu \) is poisson ratio, \( \sigma_r(D) \) is radial stress, \( \sigma_\theta(D) \) and \( \sigma_\phi(D) \) is tangential stress, \( \varepsilon_\theta \) is strain caused by temperature, \( \varepsilon_\phi \) is strain caused by irradiation, \( D' \) is reference variable of fast neutron fluence, \( E \) is elasticity modulus of graphite after irradiation, \( G(D) = (S - A_0) E e^{-SD} - (R - A_0) E e^{-RD} \) (eq. 8)

\[
S = 1.5A_0 + K(T)E - \sqrt{\frac{1}{4} (1.5A_0 + K(T)E) + EK(T)A_0}
\]

(eq. 9)

\[
R = 1.5A_0 + K(T)E + \sqrt{\frac{1}{4} (1.5A_0 + K(T)E)^2 + EK(T)A_0}
\]

(eq. 10)

Where, \( K(T) \) is creep coefficient, and \( A_0 \) is transient creep coefficient, \( A_0 = 2 \times 10^{-20} EDN \).

Plug the following boundary conditions into equation 5,

\[
\sigma_r(D) = 0, \text{ when } r = b
\]

\[
u_r(D) = 0, \text{ when } r = 0
\]

Then, the following equations are got,

\[
\sigma_r(D) = \frac{2}{1 - \mu} \left[ \beta f_r(r, D, D') - \frac{1}{r} f_r(r, D, D') \right]
\]

\[
\sigma_\theta(D) = \sigma_\phi(D) = \frac{2}{1 - \mu} \left[ \beta f_r(r, D, D') + \frac{1}{2r} f_r(r, D, D') \right]
\]

(eq. 11)

Where,

\[
f_r(r, D, D') = \int_0^D G(D - D') \left( \frac{\partial (\varepsilon_\theta(D) + \varepsilon_\phi(D))}{\partial D} \right)_r dD'
\]

(eq. 12)

\[
\beta_1 = \beta_2 = \frac{1}{b^2}
\]

(eq. 13)

Where, \( \varepsilon_\theta(D) \) and \( \varepsilon_\phi(D) \) is the strain caused by temperature and irradiation respectively, their equation are followed,

\[
\varepsilon_\theta(D) = \alpha(T) T(D) + \beta(T) \Phi(D)
\]

(eq. 14)

Where, \( T \) is temperature, \( \alpha \) is coefficient of linear expansion, \( D = \int \Phi dt \) is fast neutron fluence, \( \Phi \) is neutron flux, \( A \) and \( B \) is constant coefficient, their equation are followed,

\[
A(T) = \frac{1}{3} \left( 0.11 - 7 \times 10^{-3}T \right) \left( 5.7 - 6 \times 10^{-3}T \right)
\]

\[
B(T) = 2 \times 6 \times 10^{-3} T - 5.7
\]

(eq. 15)

As mentioned before, graphite material is isotropic and fragile. To analyze its failure probability, Local Stress Method (LSM) can be used. The theoretical foundation of LSM is Weibull statistical distribution and the weakest link theory. In LSM, the fracture volume and stress distribution are...
considered. Weibull stress is considered to be the driving force of the crack formation, and Weibull stress is determined by the double parameters of Weibull distribution with $m$ and $\sigma_0$. According to LSM, the failure probability of spherical fuel element is calculated by following equation\textsuperscript{[10-12]},

$$FR = 1 - \exp\left[\frac{1}{V_0} \int_{V_0}^{V_f} \left(\frac{\sigma_{\text{eff}}}{\sigma_0}\right)^m dV\right]$$  \hspace{1cm} (eq. 16)

Where, $m$ is parameters of weibull, $\sigma_0$ is allowable stress of graphite materials, $\sigma_{\text{eff}}$ is stress, $V_f$ is volume of control zone, $V_0$ is reference volume.

### III CALCULATION AND ANALYSIS

To calculate the temperature and failure probability of the spherical fuel element, the model, which is mentioned in chapter II, is built up with MATLAB. MATLAB is business mathematical software developed by the American MathWorks Inc. It has a wide application in data analysis and numerical calculation.

The parameters of graphite material are followed in Table 1.

Table 1: Parameters of graphite material.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.15</td>
</tr>
<tr>
<td>$H$ (W/cm$^2$C)</td>
<td>0.15</td>
</tr>
<tr>
<td>$\lambda$ (W/cm $^\circ$ C)</td>
<td>0.35</td>
</tr>
<tr>
<td>$\alpha$ (1/$^\circ$ C)</td>
<td>6x10^-6</td>
</tr>
<tr>
<td>$E$ (N/cm$^2$)</td>
<td>9.8x10^-5</td>
</tr>
<tr>
<td>$m$</td>
<td>9</td>
</tr>
<tr>
<td>$\sigma_0$ (MPa)</td>
<td>16.7</td>
</tr>
</tbody>
</table>

Power density is not the same in different position of the reactor core. Similarly, the temperature of helium and the fast neutron fluence are varied in different places. These parameters variation tendency in different position are shown in Fig 3 and Fig 4.

![Fig. 3: The power density and helium temperature in different position.](image1)

![Fig. 4: The fast neutron fluence in different position.](image2)

According to the description above, the calculating parameters are selected as Table 2.

Table 2: Calculating parameters.

<table>
<thead>
<tr>
<th>Height of core</th>
<th>$q$ (kw/fuel element)</th>
<th>$D$ ($\times 10^{21}$ EDN)</th>
<th>$T_g$ ($^\circ$ C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>550</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>650</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>750</td>
<td></td>
</tr>
</tbody>
</table>
III.A. Temperature Calculation and Analysis

In this study, one spherical fuel element located in the bottom of core is selected and the parameters in Fig 2 are used. The temperature is calculated respectively according to model 1 and model 2. The result is shown as Fig 5, Fig 6 and Fig 7.

In the result of calculation by model 2, the highest temperature is 892°C, located in the centre of the equivalent sphere. The temperature between the centre and the surface of the equivalent sphere is not in great difference. Because the reason is that, in model 2, heat is not generated by only one source. The heat source is well-distributed in comparison of model 1. The model 2 is more in line with the actual situation. It shows that, in the fuel zone of the spherical fuel element, the temperature distribution is more balanced. The lowest temperature calculated in model 2 is 843°C, located at the surface of the spherical fuel element. The temperature distribution in the fuel-free shell is linear. It is attributed to the isotropic of graphite material and this result is applicable of the actual situation.

The calculated temperature in model 2 is higher than model 1, nearly 30°C. This result also shows that, the assumption of model 1 is conservative.

The maximum temperature of spherical fuel elements in different height of core is also calculated. The result is shown in Fig 8.

It shows that, for this spherical fuel element, the temperature is decreased with the increase of radius. In the result of calculation by model 1, the highest temperature is 868°C, located in the centre of the spherical fuel element, and the lowest temperature is 809°C, located at the surface of the spherical fuel element.

![Fig. 5: The temperature distribution in spherical fuel element calculated by model 1](image)

![Fig. 6: The temperature distribution in equivalent sphere calculated by model 2](image)

![Fig. 7: The temperature distribution in fuel-free shell calculated by model 2](image)

![Fig. 8: The temperature of spherical fuel elements in different height of core](image)

It shows that, the maximum temperature is less than 1100°C. This value is below limitation of the design value.

III.B. Failure Probability Calculation and Analysis

Failure probability is calculated by model 1 and model 2 respectively. The result is shown in Table 3.

<table>
<thead>
<tr>
<th>$q$ (kw/fuel element)</th>
<th>$D \times 10^{21}$ EDN</th>
<th>$T^*_g$ (°C)</th>
<th>$FR$ (model 1)</th>
<th>$FR$ (model 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>250</td>
<td>$5.67 \times 10^{-22}$</td>
<td>$1.20 \times 10^{-18}$</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>350</td>
<td>$2.94 \times 10^{-14}$</td>
<td>$3.50 \times 10^{-11}$</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>450</td>
<td>$5.64 \times 10^{-12}$</td>
<td>$6.90 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
In comparison of the result of model 1, the failure probability calculated by model 2 is larger, because the temperature and stress calculated by model 2 are larger.

At different position, the power density and fast neutron fluence have a great influence on the failure probability. At the top of core, large power density and low fast neutron fluence are existed. In this case, the failure probability is very low. Close to the bottom of core, the failure probability increases with the fast neutron fluence increasing. At the bottom of core, the failure probability decreases because the power density is at the lowest level.

The calculated failure probability in maximum is no more than $10^{-7}$. In addition, in actual HTR-PM, some parameters are less than the ones shown in Table 2. Therefore, under the condition of normal operation of the reactor, the spherical fuel element is safe and effective.

IV CONCLUSION

In HTR-PM, spherical fuel elements with TRISO coated particles are used. The spherical fuel elements serve as reactor fuel and the second barrier preventing the radiation leaking, thus, the safety of these elements is very important. In the paper, two different models of spherical fuel elements are built up. The temperature distribution is analyzed and the failure probability is calculated. Based on the results, the maximum temperature is less than 1100°C. This value is below limitation of the design value.

The calculated failure probability in maximum is no more than $10^{-7}$. As a consequence, the spherical fuel element is safe and effective under the reactor operation conditions.

REFERENCES