Separate Effects Tests to Determine the Pressure Drop over Packed Beds in the PBMR HPTU Test Facility

CG du Toit, PG Rousseau
North-West University
Private Bag X6001, Potchefstroom 2520, South Africa
phone: +27-18-2991320, jat.dutoit@nwu.ac.za

Abstract – In this study experiments were conducted in the PBMR HPTU test facility on a small scale randomly packed cylindrical bed and a specific annular pebble bed in an effort to determine the impact of the wall effects. Tests were also conducted on test sections with structured BCC packings in an attempt to isolate the effect of porosity. The pebbles were mounted on cables and the required porosities were obtained by varying the distance between the pebbles. The required Reynolds numbers ranging between 1000 and 50000 were obtained by varying the system pressure. In the execution of the tests and the data reduction considerable care was taken to obtain good repeatability and to account for the uncertainties due to statistical variance, instrument accuracy and drift. Evaluation of the results has shown that the wall effect is negligible and that the well-known KTA correlation derived for cylindrical beds may thus be used to determine the pressure drop over the annular packed bed. The results have also shown that porosity is not the only characteristic of the packing structure that influences the pressure drop, but that amongst others the type of packing also plays an important role.

I. INTRODUCTION

The proper understanding of the relationship between the porous structure of a packed bed and the associated thermal-fluid phenomena is of utmost importance for the design, simulation and operation of systems containing packed beds. It was with this in mind that the PBMR SOC (Ltd.) developed the High Pressure Test Unit (HPTU) in cooperation with M-Tech Industrial (Pty.) Ltd. and the North-West University in South Africa to investigate amongst others the pressure drop though a packed pebble bed [1].

The pressure drop over a packed bed is one of the primary phenomena that determine the performance of a packed bed. Many tests have been conducted over the years and many correlations have been proposed [2],[3] for the friction factor used to predict the pressure drop. A perusal of the correlations show that they mainly fall in one of two categories, i.e. the Ergun-type after the correlation proposed by Ergun [4], and the Carman-type after the correlation proposed by Carman [5].

The German Nuclear Safety Standards Commission made a thorough investigation of available correlations and data to develop a friction factor correlation for packed pebble beds [6]. The following criteria were applied to the correlations and data, i.e. (i) the wall effect had to be negligible, (ii) the bulk porosity had to be negligible, (iii) bed length to particle diameter ratio had to be larger than four, and (iv) the particle diameter had to be larger than 1 mm. From the regression analysis performed on the semi-empirical data a Carman-type correlation was proposed. This correlation is widely used to predict the pressure drop over pebble bed reactors.

The purpose of the pressure drop tests performed in the HPTU were to check the validity of the KTA correlation for the PBMR cylindrical and annular configurations, as well as the applicability of the correlation for the porosities typically found in the near wall region [1].

In this paper the tests that were conducted using the HPTU to determine the friction factor for various configurations will be described. The
measured data, the reduction of the data to derive the friction factor and the associated uncertainties will be discussed. The results for the friction factor are evaluated and compared with values predicted by the KTA and other correlations.

II. THEORETICAL OVERVIEW

The Darcy-Weisbach approach for pipe flow is the most common method found in the literature to describe the pressure drop \( \Delta p \) over packed beds [7]. Accounting for the relationship between the hydraulic diameter and the particle diameter leads to:

\[
\Delta p = \psi \frac{\rho V^2}{2} \frac{1 - \varepsilon}{\varepsilon^3} \frac{L}{d_p}
\]

(eq. 1)

with

\[
\psi = f \left( \frac{\rho V_d d_p}{(1 - \varepsilon) \mu} \right)
\]

(eq. 2)

\[
= f \left( Re_m \right)
\]

In eq. (2) \( \psi \) is the friction factor, \( \rho \) the density of the fluid, \( V_d \) the superficial velocity, \( \varepsilon \) the average porosity of the bed, \( L \) the length of the bed, \( d_p \) the particle diameter, \( \mu \) the dynamic viscosity of the fluid and \( Re_m \) the modified Reynolds number.

The KTA correlation for the friction factor is given as [6]:

\[
\psi = \frac{320}{Re_m} + \frac{6}{Re_m^{0.1}}
\]

(eq. 3)

Eq. (3) is valid for \( 10 \leq Re_m \leq 10^5 \), \( 0.366 < \varepsilon < 0.43 \), \( L/d_p > 5 \) and cylinder diameter to particle diameter ratios larger than a limiting line given in graphical form only. The uncertainty range for the correlation is given as \( \pm 15\% \) within the scope of application with a confidence level of 95%.

III. EXPERIMENTAL SET-UP

Five test sections were manufactured and mounted one at a time in the HPTU facility. The test sections were mounted in the pressure vessel of the HPTU plant as shown schematically in Fig. 1. Note that the braiding loop was closed off during the pressure drop tests. The system pressure could be varied between 1 bar and 50 bar. This made it possible to vary the superficial Reynolds number between 1000 and 50000. The working fluid was nitrogen. The system was supplied with the required pressure from high pressure nitrogen banks in an auxiliary bay. A more detailed description of the system can be found in Rousseau and Van Staden [1] and Van der Walt [8].

III.A. Test Sections and Layout

Three test sections with homogeneous porosities were manufactured in an effort to isolate the effect of porosity and evaluate the validity of the KTA correlation in the near wall region [1]. A body-centered cubic (BCC) packing structure was chosen. The porosities were obtained by mounting the acrylic spheres with a diameter of 28.575 mm on thin cables with spacers in between of different lengths to vary the distance between the spheres. The wall effect was eliminated by fixing appropriately cut spheres to the walls. The test sections had a cross section of 300 mm x 300 mm. The lengths and porosities of the three beds are summarized in Table 1 (including and not including the spacers).

Table 1: Lengths and porosities of homogeneous porosity packed beds.

<table>
<thead>
<tr>
<th>Test Section</th>
<th>Length mm</th>
<th>Porosity including spacers</th>
<th>Porosity not including spacers</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDTS036</td>
<td>741.1</td>
<td>0.352</td>
<td>0.369</td>
</tr>
<tr>
<td>PDTS039</td>
<td>756.5</td>
<td>0.389</td>
<td>0.407</td>
</tr>
<tr>
<td>PDTS045</td>
<td>781.7</td>
<td>0.448</td>
<td>0.465</td>
</tr>
</tbody>
</table>

The PDTS036, PDTS039 and PDTS045 beds consisted of approximately 3445, 3307 and 3080 spheres respectively. It should be remembered that a closely packed BCC bed has a porosity of 0.32. In the test sections none of the spheres were therefore in contact with any other sphere. A schematic representation and pictures of a PDTS test section, including the layout of the pressure tapping points is shown in Fig. 2.

Two randomly packed beds with cylindrical (small cylindrical pebble bed) and annular (small annular pebble bed) configurations respectively were also manufactured. The radial dimensions of the beds were chosen to be one-tenth of the dimensions of the actual pebble bed reactor cores that were considered during the course of the PBMR project. In both beds 6 mm steel ball bearings were used as particles.

The small cylindrical pebble bed (SCPB) had an outer diameter of 368.9 mm, a length of 178.2 mm, and an average porosity of 0.393 and contained approximately 104400 particles.
Fig. 1: Layout of HPTU plant [8]

Fig. 2: Schematic representation of a PDTS test section.
III.B. Uncertainty analysis

An uncertainty analysis was performed for each measured variable and an error propagation analysis was performed to obtain the uncertainty for each derived variable.

Two orifice stations were used to determine the mass flow rate through the system (see flow measurement devices in Fig. 1). To obtain the mass flow rate the pressure differential over each orifice plate, the absolute static pressure before each orifice plate and the temperature after each orifice plate were measured. In order to be able to determine the friction factor of a packed bed the pressure differential over the relevant packed bed, the absolute static pressure at the inlet of the test section and the temperature upstream of the inlet of the test section were measured. Three pressure differential transmitters were used to measure the pressure drops over the beds. The instruments had ranges of 0 – 10 mbar, 0 – 100 mbar and 0 – 500 mbar with instrument uncertainties of ±0.015 mbar, ±0.05 mbar and ±0.25 mbar respectively. For each measurement the most accurate instrument was used.

For each measured variable \( \phi \), the total standard uncertainty \( u_\phi(\phi) \) was calculated by combining the statistical uncertainty \( u_\sigma(\phi) \), instrument uncertainty \( u_{in}(\phi) \) and the drift uncertainty \( u_{dr}(\phi) \) as follows:

\[
 u_\phi(\phi) = \sqrt{u_\sigma(\phi)^2 + u_{in}(\phi)^2 + u_{dr}(\phi)^2} 
\]  
(eq. 4)

All instruments were calibrated before and after the test period using secondary standard instruments calibrated by an accredited laboratory. The instrument uncertainty was taken as the maximum of the calibrated and un-calibrated instrument (provided by the manufacturer) uncertainties. All uncertainties were converted to standard uncertainties where necessary. The drift uncertainties were obtained by comparing the calibrations before and after the test period.

In the case of the geometric variables that were not measured explicitly the manufacturing tolerances were converted to standard uncertainties using a coverage factor of \( 3 \).

The derived variables that were obtained from the measured variables were the densities, dynamic viscosities, mass flow rate, modified Reynolds number and the friction factor for the packed bed. The required densities and dynamic viscosities for the nitrogen were calculated from the relevant pressure and temperature using correlations that were fitted to the REFPROP data [9] for the density and the dynamic viscosity. The maximum difference between the correlations and the original REFPROP data was taken as the standard uncertainties associated with the correlations and accounted for in the relevant calculations.

The total standard uncertainty for a derived variable \( \phi = f(\phi_1, \phi_2, \ldots, \phi_n) \) was obtained using an error propagation analysis [10]:

\[
 u_\phi(\phi) = \sqrt{\sum_{j=1}^{n} \left( \frac{\partial f}{\partial \phi_j} u_{\phi_j}(\phi) \right)^2} 
\]  
(eq. 5)

All the relevant functional relationships are such that the sensitivity coefficients could be calculated using analytical expressions and no perturbation analyses were required. The final expanded uncertainty \( u_\varphi(\phi) \) for the friction factor with a confidence level of 95% was obtained by using a coverage factor of two, thus:
Fig. 4: Friction factor $\psi$ as a function of the modified Reynolds number $Re_m$ for all the tests.

\[ u_f(\phi) = 2u_r(\phi) \]  

(eq. 6)

III. RESULTS

Four sets tests were performed for each packed bed to check for repeatability. The maximum difference between the values obtained for the friction factor for the homogeneous PDTS test sections was 3%, whilst the maximum difference between the values obtained for the SCPB and SAPB test sections was 4.5%. This is illustrated in Fig. 4 by how closely the data points are grouped for each test section at the various Reynolds numbers.

III.A. Friction factors for all tests

The friction factor as a function of modified Reynolds number for all the tests are shown in Fig. 4. In the case of the PDTS test sections the porosity that was used, was the porosity without accounting for the spacers (effective porosity as opposed to the true porosity accounting for the spacers). It was argued that the contribution of the spacers to the pressure drop is negligible compared to that of the spheres. Had the true porosity been accounted for the friction factors would have been approximately 15% smaller and the associated modified Reynolds number approximately 3% lower.

We will first consider the results for the SCPB and SAPB test sections. In the case of the SCPB tests the maximum final uncertainty is 8.21%, whilst for the SAPB test the maximum final uncertainty is 11.05%.

A study of Fig. 4 reveals that there is little difference between the friction factor values for the SCPB and SAPB test sections. There is also only a small difference between the friction factor values for the SCPB and the SAPB and the values predicted by the KTA correlation. Both sets of results also fall within the uncertainty band of the KTA correlation indicated by the dashed lines. The conclusions that can be drawn from the results are that the KTA correlation is confirmed by the experimental results and that the KTA correlation can also be applied in the case of this specific annular configuration to calculate the pressure drop over the packed bed. It implies that the walls had a negligible effect on the pressure drop through the annular packed bed. This is interesting considering the fact that with an annular width of 14 sphere diameters the wall affected region covers approximately 85% of the annular packed bed.

Secondly we consider the experimental results for the PDTS test sections. The maximum final uncertainties were 7.0%, 7.01% and 7.65% for the PDTS036, PDTS039 and PDTS045 test sections respectively.

When looking at the relevant friction factor values in Fig. 4 it is observed that the values for the three homogeneous porosity test sections do not collapse onto a single curve. This suggests that it is not only the average porosity that has an influence
Fig. 5: Experimental results for the friction factor compared the values predicted by various correlations.

on the friction factor. This is confirmed when the experimental values of the friction factor are compared with those predicted by the KTA correlation. The experimental results follow a trend similar to that of the KTA correlation, but at a much steeper gradient and there is a marked difference between the experimental results and the predicted values. It is only at the lowest Reynolds number that the experimental results fall within the uncertainty band of the KTA correlation. Clearly the friction factors for the structured packings are consistently lower than that of the random packings and the friction factor is inversely proportional to the porosity.

III.B. Comparison with other correlations

In Fig. 5 the comparison between the experimental values obtained for the friction factor are compared with the values predicted by the KTA correlation [6], Ergun correlation [4], Eisfeld and Schnitzlein correlation [11] and the correlation given by Wentz and Thodos [12]. The purpose of the comparison is to attempt to gain an understanding of the difference between the friction factors for the unstructured and structured packed beds.

The Ergun correlation [4] is based on pressure drop measurements for flow through beds packed with particles of various shapes and sizes, such as various sized spheres, sand and pulverized coke. The Ergun correlation [4] is given as:

$$\psi = \frac{300}{Re_w} + 3.5$$  \hspace{1cm} (eq. 7)

The range for the original experimental data on which the correlation is based was $0.36 \leq \varepsilon \leq 0.4$ and $1 \leq Re_p \leq 2500$ with $Re_p = Re_w (1 - \varepsilon)$.

Eisfeld and Schnitzlein [11] investigated the influence of the cylinder wall on the pressure drop with the aim of establishing which existing correlations are valid when the wall effects are not negligible. They studied 24 correlations with more than 2300 data points. They found the correlation proposed by Reichelt [13] to be the most suitable and refined the correlation to fit the data. The proposed correlation is an Ergun-type. For packed beds consisting of spherical particles the Eisfeld and Schnitzlein correlation [11] is given as:

$$\psi = \frac{308A_w^2}{Re_w} + \frac{2A_w}{B_w}$$  \hspace{1cm} (eq. 8)

with

$$A_w = 1 + \frac{2}{3(D/d_p)(1 - \varepsilon)}$$  \hspace{1cm} (eq. 9)

$$B_w = \left[ 1.15 \left( \frac{d_p}{D} \right)^2 + 0.87 \right]^2$$  \hspace{1cm} (eq. 10)
The correlation is valid for $0.33 < \varepsilon < 0.882$, $0.01 \leq \text{Re}_n \leq 17635$ and $1.624 \leq D/d_p \leq 250$ where $D$ is the cylinder diameter. The uncertainty range for the correlation is given as ±31% within the scope of application with a confidence level of 95%.

Two curves based on the Eisfeld and Schmitzlein correlation are shown in Fig. 4, namely for $D/d_p = 4 \ (E+S-4)$ and $D/d_p = 10 \ (E+S-10)$.

Wentz and Thodos [12] measured the pressure drop over packed beds of spheres arranged in cubic, BCC and face-centred cubic (FCC) orientations. Both closely packed and distended beds were investigated for all packing arrangements. The distended beds were obtained by fixing adjacent spheres by short lengths of wire at the appropriate angles to each other. They fitted a correlation to all the data, as well as a correlation to the data for the distended beds. The Wentz and Thodos prediction shown in Fig. 5 is the correlation for the distended beds. The correlation is given as [12]:

$$\psi = \frac{0.702}{\text{Re}_w^{0.01} - 1.2} \quad (\text{eq. 11})$$

The range for the experimental data on which the correlation is based was $3860 \leq \text{Re}_n \leq 64920$ and $0.615 \leq \varepsilon \leq 0.882$.

A study of Fig. 5 first of all reveals that one can make a distinction between the results for the friction factor for $\text{Re}_n \leq 300$ and those for $\text{Re}_n \geq 300$. When $\text{Re}_n \leq 300$ the friction factors predicted by the KTA and Ergun correlations are in close agreement. In both cases the effect of the wall is assumed to be negligible. The results for the Eisfeld and Schmitzlein correlation indicate the contribution to the friction by the wall when the wall effect cannot be neglected. However for $\text{Re}_n \geq 300$ the results for the Eisfeld and Schmitzlein correlation indicated that the effect of the wall becomes negligible. It can be concluded that for $\text{Re}_n \leq 300$ the pressure loss is dominated by viscous effects.

For $\text{Re}_n \geq 300$ the values predicted by the Ergun and KTA correlations move apart with the values predicted by the Ergun correlation being larger than those predicted by the KTA correlation. It should be remembered that the KTA correlation was derived from the results for packed beds consisting of uniformly sized spheres. In contrast the Ergun correlation was derived from the results for packed beds consisting of particles with various shapes and sizes.

Although the porosities of the distended beds investigated by Wentz and Thodos [12] were larger than the porosities of the PDTS test sections, the values predicted by the Wentz and Thodos correlation are larger than the values obtained in the current tests. A possible explanation is that the way in which Wentz and Thodos fixed adjacent spheres to each other resulted in the wires not necessarily being aligned with the flow. This could have resulted in added resistance. In the current tests the spacers were aligned with the main flow direction and its effect on the pressure drop should therefore be very small.

The differences between the various results for the friction factor for $\text{Re}_n \geq 300$ suggest that the pressure drop is dominated by inertial effects and that the average porosity alone is not sufficient to determine the friction factor. Other parameters that characterize the porous structure of a packed bed such as the specific packing structure should also be accounted for.

It is therefore recommended that an in depth investigation be performed to describe the friction factor for the flow through packed beds for modified Reynolds numbers larger than 300.

IV. CONCLUSIONS

In this paper the tests that were conducted using the HPTU to determine the friction factor for various configurations were described. The measured data, the reduction of the data to derive the friction factor and the associated uncertainties were discussed. The results for the friction factor were evaluated and compared with values predicted by the KTA and other correlations.

From the results obtained from the SCPB and SABP test sections it was concluded that the KTA correlation could be used to predict the pressure drop over both the cylindrical and annular packed bed configurations considered by the PBMR SOC (Ltd.).

The results obtained for the friction factor from the PDTS test sections, and the comparison with the values for the friction factor predicted by selected correlations, show that for modified Reynolds numbers lower than 300 the pressure drop over the bed is dominated by viscous forces. Under these circumstances the effect of the container wall can be important. For modified Reynolds numbers larger than the 300 the pressure drop is dominated by inertial forces. In this case the effect of the wall is negligible and the porous structure of the packed bed plays an important role. It is recommended that an in depth investigation be conducted to describe the friction factor for flows through packed beds for modified Reynolds numbers larger than 300.
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