X-rays and synchrotron radiation
Synchrotron Radiation: electromagnetic radiation
Electromagnetic spectrum
Mechanism of production of e.m. radiation

• Emission from accelerated electric charges

• Emission as effect of quantum transitions
Emission as effect of quantum transitions

\[ h\nu = E_{\text{high}} - E_{\text{low}} \]
Emission from accelerated charges

Oscillating motion

Average linear deceleration

Relativistic circular motion

Broadcasting antenna

Bremsstrahlung in x-ray tubes

Synchrotron radiation

Tecniche di caratterizzazione con luce di sincrotrone
Emission from accelerated charges: dipole radiation

Radiated power

\[ P \propto \cos^2 \theta \]
Standard x-ray tubes

The electrons are produced by thermoionic effect from a tungsten filament heated by an electric current. The filament is the cathode of the tube.

A high voltage potentials is between the cathode and the a water cooled metallic anode.

The current and the high voltage can be varied independently. The intensity limitation is set by the maximum power a cooled metal anode can withstand (few kW)

Typical accelerating voltages: 100 kV
Typical current : 1 mA to 1 A

Different anode materials: Cr, Cu, Mo, W,..
Emission spectrum

- Characteristic radiation
- Bremsstrahlung radiation

Graph showing emission spectrum with energy on the x-axis and log intensity on the y-axis.
Bremsstrahlung radiation
electromagnetic radiation emitted when \textbf{relativistic} \textbf{charged particles} are subject to an acceleration perpendicular to their velocity
Storage rings

5 main components
Electrons are generated by thermoionic emission from a hot filament in an electron gun.

The electrons are accelerated using a Linac to about 100 MeV.

A regular supply of electrons is required, as they are always being lost in the machine, due to collisions with residual gas particles in the storage ring.
Electrons are injected from the linac and further accelerated.

They may either be accelerated to the energy of the electrons in the main storage ring, or (less commonly, especially for modern facilities) to a somewhat lower energy.
Electrons are injected from the booster periodically so that the specified current is maintained. This is done when the current drops to about $1 - \frac{1}{e}\approx 70\%$ or more often in case of top up mode.

The storage ring contains the electrons and maintains them on a closed path by the use of an array of magnets, commonly referred to as the ‘magnet lattice’ of the ring.

The electrons have kinetic energies measured in GeV, and their velocities are highly relativistic, that is, only very marginally less than the velocity of light.
Storage ring: magnet lattice

Bending, dipole-magnets
They cause the electrons to change their path and thereby follow a close path

Quadrupole-magnets
They are used to focus the electron beam and for Coulomb repulsion between electrons

Sextupole-magnets
They correct the chromatic aberration that arise from focusing by the quadrupoles
The ring has a structure consisting of arced sections containing bending magnets (BMs) and straight sections used for insertions devices (IDs), which generates the most intense SR.

The BMs, used to deflect the electrons round the arced sections that connect the straight sections are also often used to provide BM radiation – although their brilliance is significantly lower than that produced by IDs, even monochromated BM-radiation is still orders of magnitude more intense than that can be provided by laboratory-based sources.
Tecniche di caratterizzazione con luce di sincrotrone
The kinetic energy of the electrons dissipated due to emission of radiation at BMs and IDs must be replenished before they spiral into the inner wall of the storage ring.

This is achieved by giving them a small boost at every turn as they pass through a radio-frequency cavity (klystron).
They run off tangentially to the storage ring, along the axes of the IDs and tangentially to BMs.
• 47 SR research facilities
• 23 countries
• more than 30000 scientists and engineers (including thousand of students)
First observation of synchrotron radiation on the 24 April 1947 from the 70 MeV electron synchrotron at General Electric
History of synchrotron radiation - 2

• **First generation** (60s): parasitic facilities. SR was seen as an unwanted but unavoidable loss of energy in accelerators designed for high-energy or nuclear physics experiments.

• **Second generation** (80s): dedicated machines for synchrotron radiation. Main source: bending magnets.

• **Third generation** (90’s): dedicated machines designed for optimal brilliance with insertion devices such as wigglers and undulators. ESRF (Grenoble, France) was the first third generation synchrotron source. It start user operation in 1994. Elettra is also a third generation synchrotron.

• **Fourth generation** (00’s): Free electron lasers (Fermi, LCLS), Improved performances with regards to coherence and brilliance of x-rays.
Evolution of performances
The energy $\mathcal{E}_e$ of an electron at speed $\nu$ is

$$\mathcal{E}_e = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{\nu}{c}\right)^2}}$$

$\gamma = \frac{1}{\sqrt{1 - \left(\frac{\nu}{c}\right)^2}}$ Lorentz factor

$$\beta = \frac{\nu}{c}$$

$m_0 c^2$ rest mass energy of the electron $= 511$ keV
If $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, then

$$\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

For typical values of $\gamma$ of $10^4$, $1/\gamma^2$ is very small, so we can retain the first two terms of the Taylor expansion, and then obtain:

$$\beta \approx 1 - \frac{1}{2\gamma^2}$$
An electron moving in a magnetic field radiates energy in form of electromagnetic radiation. At relativistic velocities it is synchrotron radiation.

\[ p = mv \]

An electron of momentum \( p = mv \) moving in a constant magnetic field \( B \) experiences the Lorentz force \( F = \frac{dp}{dt} = ev \times B \). In response to this force the electron accelerates and moves in a circular orbit in a plane perpendicular to \( B \).

The Lorentz force, being perpendicular to the motion, does no work and cannot change the energy of the electrons, but does cause a centripetal acceleration that changes the direction of the velocity.
Considering the relativistic formulae:

\[ \mathbf{p} = \gamma m_0 \mathbf{v} \]

\[ \mathbf{F} = \frac{d}{dt} (\gamma m_0 \mathbf{v}) = e \mathbf{v} \times \mathbf{B} \quad \text{They are perpendicular} \]

\[ \gamma m_0 \frac{d}{dt} \mathbf{v} = e \mathbf{v} \mathbf{B} \quad \text{Centripetal acceleration} \]

\[ \gamma m_0 \frac{v^2}{\rho} = e \mathbf{v} \mathbf{B} \quad \text{Orbital radius} \]

Since \( v \approx c \),

\[ c^2 \]
Equations of motion - 3

\[ \rho = \frac{\gamma m_0 c}{eB} \]

Since \( \gamma = \frac{\varepsilon}{m_0 c^2} \)

\[ \rho = \frac{\varepsilon}{ceB} \]

In practical units:

\[ \rho [m] = 3.3 \frac{\varepsilon [GeV]}{B [T]} \]

- As typical magnetic field strengths of bending magnets are 1 Tesla and storage rings electron energies are normally of the order of few GeV, the bending radius is typically a few meters
Tecniche di caratterizzazione con luce di sincrotrone

ESRF : 6 GeV (845 m)

APS : 7 GeV (1104 m)

SPring8 : 8 GeV (1435 m)
Power radiated by a relativistic electron forced to move along a circular orbit, with radius of curvature $\rho$

$$P_e = \int \int P(\lambda, \psi) d\lambda d\psi = \frac{2}{3} \frac{e^2 c}{\rho^2} \left(\frac{\mathcal{E}}{m_0 c^2}\right)^4$$

Schwinger’s formula

$\lambda$: wavelength of the emitted radiation

$\psi$: vertical half opening angle perpendicular to the orbit plane

$P(\lambda, \psi)$: power radiated by an electron in a unit wavelength centered at $\lambda$, and in a unit vertical angle centered at $\psi$

$\mathcal{E}$: electron energy

$c$: speed of light

$m_0 c^2$: electron rest mass energy

The radiation produced by proton accelerators is negligible.
Since $\mathcal{E} = \gamma m_0 c^2$

\[ P_e = \frac{2}{3} \frac{e^2 c}{\rho^2} \gamma^4 \]

High energy storage rings produces large radiated power

In practical units, the radiated power from a circular arc of length $L[m]$:

Energy loss

The energy lost per turn by the charged particle, taking into account that the revolution time $t \sim 2\pi \rho / c$:

$$\Delta \mathcal{E} = P_e \cdot t$$

$$\Rightarrow \Delta \mathcal{E} \sim \frac{4\pi e^2}{3} \frac{\rho}{\gamma^4}$$
Radio frequency and time structure - 1

- The kinetic energy of the electrons dissipated due to emission must be replenished before they spiral into the inner wall of the storage ring.
- This is achieved by radio frequency cavities (klystron).

A periodic electric field is applied in the direction of the electrons.
• The RF fields have an accelerating effect only during one half of their period and a decelerating action during the other half: so the RF is effective in restoring the electron energy only for one half of the time.

• Additional considerations have to be performed, regarding the stability of the electron
• On average, the electrons require a certain boost in order to maintain them on a stable path, given by an amount $eV_{\text{ref}}$

• **Slow electrons** (those that have dissipated more energy) will arrive in A finding an electric field higher than that found by synchronous electrons otherwise they will continue to lose energy with respect to them. In the next turn they will arrive later, and after some turns they will enter the RF during the decelerating semi-period and will be lost.

• Viceversa, **fast electrons** will arrive in B before the synchronous ones finding a lower electric field, otherwise their energy will increase with respect to the synchronous electrons.
• Electrons outside 5-10% the reference voltage will not gain the correct energy and will be lost
• As a consequence the electrons in the storage ring are grouped in bunches with time lengths that are typically 5–10 % of the RF period
• The radiation appears in pulses
Along the storage ring many bunches can be distributed. The time interval between them is an integer multiple of the RF period.

The maximum separation between two pulses is obtained in the single bunch mode. In this case the time interval is equal to the period of revolution (μs).

For more bunches, the minimum possible time interval between bunches is equal to the RF period.

The filling of bunches in a machine can be completely controlled.

The total current depends on the number of filled bunches. The current is lower when few bunches are filled, because the total amount of current per bunch is limited.
Coulomb repulsion in electron bunches

- Because of the relativistic length contraction, the distances ‘seen’ by electrons moving together in the same frame of reference is $\gamma$ times larger than in the laboratory frame of reference.
- The Coulomb force between the electrons are $1/\gamma^2$ weaker.
- “Beam stiffness” increases with energy.
- Nonetheless, the beam must be manipulated into the right dimensions and positions using quadrupole and hexapole magnets.
In the moving frame of the electron (\(v \ll c\)) the power emitted by an accelerated particle has a characteristic two-lobe distribution around the direction of the acceleration.

The dependence of the radiated power \(P\), on the angle relative to the direction of the acceleration, \(\theta\) is given by:

\[
P \propto \cos^2 \theta
\]
In the laboratory frame of reference \((v \sim c)\) all the emitted power is beamed into a narrow cone in the direction of motion.

All the forward power is radiated in a beam of angle

\[
\theta \approx \frac{1}{\gamma^2}
\]
• The collimation of synchrotron radiation is a direct consequence of the relativistic speed of the electrons
• The collimation conserves energy: the emission found in the electron frame is now concentrated in a small cone.
• This affects a fundamental figure of merit for light sources: the “brightness”. (The brightness is proportional to the emitted flux divided by the angular spread and by the source size.)
• Very high fluxes on very small area also at distances of tens of meters from the storage rings.
Emission from a bending magnet

- The natural collimation $\psi$ is preserved only in the vertical direction.
- The horizontal collimation is lost because the electrons move along a circular orbit emitting the radiation along the tangent.
- The radiation is collected through a horizontal slit (S) of width $w$ and distance $D$ from the electron orbit.
- This corresponds to an angular collection angle $\Delta \theta = w/D \gg \psi$. 
Synchrotron light emission occurs primarily at high energy in the x-ray range.
• The time of emission of radiation is given by the time the electron runs at speed $v$ along the arc from 1 to 2
• However the radiation emitted at 1 will reach the observer with a delay given by the fact that the radiation must travel (at speed $c$) to the observer
The pulse duration $\Delta \tau$:

$$\Delta \tau = \frac{\text{arc length}}{\nu} - \frac{\text{radiation path}}{c}$$

$$\sigma^2/\nu = 2\sigma \sin(1/\nu)$$
Pulse duration - 4

\[ \Delta \tau \approx \frac{2\varrho}{\nu \gamma} - \frac{2\varrho}{c \gamma} \]

\[ \Delta \tau \approx \frac{2\varrho}{\gamma} \left( \frac{1}{\nu} - \frac{1}{c} \right) = \frac{2\varrho}{\gamma} \left( \frac{1}{\beta c} - \frac{1}{c} \right) = \frac{2\varrho}{\gamma c} \left( \frac{1}{\beta} - 1 \right) = \frac{2\varrho}{\gamma c} \left( \frac{1 - \beta}{\beta} \right) \]

\[ \approx \frac{2\varrho}{\gamma c} (1 - \beta) \approx \frac{2\varrho}{\gamma c} \frac{1}{2\gamma^2} \]

\[ \Delta \tau \approx \frac{\rho}{c \gamma^3} \]
Time pulse and peak of emission

• The Fourier theorem relates this pulse duration $\Delta \tau$ to a typical frequency $\nu_c$

\[
\nu_c \approx \frac{1}{2\pi \Delta \tau}
\]

\[
\nu_c \approx \frac{1}{2\pi} \frac{\rho}{c\gamma^3}
\]

$\nu_c \approx \frac{c\gamma^3}{2\pi \rho}$ \Rightarrow $E_c = h\nu_c \approx \frac{hc\gamma^3}{2\pi \rho}$

• The corresponding photon energy is high, in the x-range, primarily due to the large $\gamma$ factor.
• Using the relativistic mass $\gamma m_0$, the relation between force and acceleration is $\frac{\gamma m_0 v^2}{q} = evB$, hence $\rho = \gamma m_0 v / (eB)$ and:

$$E_c \approx \frac{hc \gamma^2 eB}{2\pi m_0 v} \approx \frac{h \gamma^2 eB}{2\pi m_0}$$
• The synchrotron has an emission that is peaked in $h\nu_c$

• In practice, the spectral distribution is broadened as it comes from many electrons the statistically oscillate around the main orbit.
Polarization

The polarization of the x-rays emerging from a storage ring depends on the line of sight.

- **Linearly polarized**
  - In-plane

- **Left circular polarized (LCP)**
  - Above plane

- **Right circular polarized (RCP)**
  - Below plane

The electrons appear to execute an elliptical orbit in clockwise/anticlockwise direction. The angular momentum is transferred to the emitted photons which are LCP/RCP. The electric field vector of the x-rays also rotates clockwise/anticlockwise around the direction of propagation. The angular momentum of the rotating electric field of the LCP/RCP photons is \(-\hbar/\hbar\). Note that the angular momentum of a photon can be transferred to systems which absorb that photon. This is an important fact for understanding (among other things) dichroism in spectroscopy.
Coherence enables to observe phenomena such as diffraction and interference.

**Pinhole diffraction**

- **Point source, one photon energy (\(h\nu\))**
  - Each point produces a diffraction pattern.
  - They are superimposed.

- **Extended source, one photon energy (\(h\nu\))**
  - Each energy within the band produces a diffraction pattern.
  - They are superimposed.

- **Point source, photon energy band (\(\Delta h\nu\))**
  - Each energy within the band produces a diffraction pattern.
  - They are superimposed.
Coherence - 2

Longitudinal or time coherence: linked to the spectral bandwidth
Lateral or space coherence: linked to the source geometry

- time and space coherent
- time coherent
- space coherent
Longitudinal coherence

Criterion for minimal longitudinal (temporal) coherence:

\[ \frac{\Delta E}{E} < 1 \]

For broadband emission (bending magnets):

- light can be monochromatized \((\Delta E/E = 10^{-4} \text{ for Si}(111))\) (lesson 5 for monochromators)

For non broadband emission (undulators):

- sources naturally longitudinally coherent (lesson 2 for undulators)
Lateral coherence

- The fraction of lateral coherent power characterizes the level of lateral coherence of the source.

\[ C = \frac{(\lambda/2)^2}{\sigma_x \sigma_y \theta_x \theta_y} \]

- All types of coherence are more difficult to achieve at high photon energies (x-rays) than at lower photon energies like visible or infrared.

- The improved geometry of the source lead not only to and increase of brightness/brilliance, but also a boost of lateral coherence.
Lateral coherence - Elettra

Elettra is fully lateral coherent down to $\lambda = 1 \text{ Å}$

Conventional absorption image

Coherence-enhanced image
Esercizi

A. Per Elettra (www.elettra.eu) calcolare:
   1. $\gamma$ per $E=2.0$ GeV e $E=2.4$ GeV
   2. Velocità degli elettroni ovvero il fattore $\beta$
   3. Calcolare la divergenza $1/\gamma$
   4. Raggio dell’orbita $\rho$

B. Per Elettra calcolare a che angolo si trova la radiazione che nel Sistema di riferimento dell’elettrone ($v<<c$) è a:
   1. $\theta = \pi/4$
   2. $\theta = \pi/3$