Synchrotron radiation: sources and figures of merit
• The primary purpose of the bending magnet is to circulate the electron beam in the storage ring in a close path
• The bending magnet is also used as source for synchrotron radiation

The angular spread of the BM radiation is a flattened cone \((2/\gamma \text{ vertically})\) with an horizontal angle equal to the angular change of the path of the electrons
Bending magnet: spectral distribution - 2

Linear-linear plot of the approximate lineshape for broadband bending magnet emission

Log-log plot of the approximate lineshape for broadband bending magnet emission
A more precise description of the photon energy distribution can be obtained by a rigorous solution of Maxwell’s equations for a relativistic electron in a uniform magnetic field

\[
F \left[ \frac{\text{photons/seconds}}{(m rad^2)(0.1\% BW)} \right] = 1.33 \times 10^{13} \mathcal{E}^2 [\text{GeV}] I [\text{A}] x^2 K_{2/3}(x/2)
\]

where \( x = \nu/\nu_{CR} \) and \( K_{2/3}(x/2) \) is a modified Bessel function.
Bending magnet: critical energy

$h_{\nu CR}$

- The critical energy is defined by saying that equal amounts of synchrotron radiation energy are emitted at photon energies lower and higher than $h_{\nu CR}$.

- The rigorous theories of the emission demonstrate a direct link between $h_{\nu c}$ and $h_{\nu CR}$:

$$h_{\nu CR} = \frac{3}{4}h_{\nu c}$$

- In practical units:

  $$h_{\nu CR}[\text{keV}] = 0.665\varepsilon^2[\text{GeV}]B[\text{T}]$$

  $$h_{\nu CR}[\text{keV}] = 2.21\varepsilon^3[\text{GeV}]/\rho[\text{m}]$$
Third generation synchrotrons are characterized by the use of insertion devices (IDs).

These are placed in the straight sections between the bending magnet arc segments
• Insertion devices are periodic magnetic structures
• (e.g. permanent magnets: NdFeB).

• Passing through such alternating magnetic field structures, electrons oscillate perpendicularly to the direction of their motion and therefore emit SR during each individual wiggle.
Effect of insertion devices:

• To shift the critical energy $h\nu_{CR}$ to higher values due to the smaller bending radius $\rho$ with respect to the bending magnets

• To increase the intensity of the radiation by a factor related to the number of wiggles induced by many poles of the magnetic structure

• To increase the spectral brightness
Insertion devices - 4
Wigglers and undulators are different from one another by the degree to which the electrons are forced to deviate from a straight path.

**Wigglers**

**Undulators**
Wigglers and undulators - 2

Wigglers

Just like a BM except:
• Larger $B \rightarrow$ higher $h\nu_{CR}$
• More bends $\rightarrow$ more power

Undulators

Different from BM:
• Shallow bends $\rightarrow$ small source
• Interference effect $\rightarrow$ highly peaked spectrum
Wigglers - 1

• Multipole magnet made up of a periodic series of magnets (N periods of length $\lambda_w$, the overall length being $L = N\lambda_w$) (typical $L = 2m$, typical $\lambda_w = 1-2 cm$)

• The maximum angular excursion from the wiggler axis is larger than the natural opening angle of radiation $2/\gamma$

• Each curve there is a cone of emission that does not interfere with the next one

• If there are N periods, the electrons will do $2N$ arcs

• The radiation is enhanced by a factor $2N$

• The spectrum from a wiggler has the same form as that of a BM
• Magnetic fields of wigglers (~3 T) are higher than those in BMs (~1 T), therefore electrons follow a curved trajectory with a smaller radius of curvature $\rho$ with respect to the one of the BM.

$B = 0.74 \text{ T}$

$B = 3 \text{ T}$
• The emitted power is similar to that of a bending magnet
• In a BM the magnetic field is constant, while in a wiggler the field strength oscillates along the ID axis, (B drop to zero exactly between magnet pairs)
• The square of the average field is \( B^2 / 2 \), where \( B \) is the maximum magnetic field strength
• The total power emitted by a wiggler is
  \[
P_w [\text{kW}] = 0.633 \varepsilon^2 [\text{GeV}] B^2 [\text{T}] L [\text{m}] I [\text{A}]\]

The amount of power radiated by a wiggler increases as the gap between the upper and the lower sets is reduced (\( B \) increases)
Power of wiggler radiation

Figure 3.17 The power of wiggler radiation. These images of a runaway failure of the beam-defining aperture in the front end of a wiggler beamline at the Swiss Light Source demonstrate the power delivered at synchrotron facilities. The aperture consists of two pairs of tungsten blocks, and is (or, rather, was) the first component in the beamline, and therefore ‘sees’ the full spectrum of radiation from the wiggler. This includes the soft x-ray component, which is very efficiently absorbed by matter, before this dangerous spectral component is removed by a carbon or diamond filter. Failure occurred because the upper block was not held sufficiently firmly in place, and dropped marginally into the beam path. The increased heat load caused the support structure, made of copper, to also become hot, which loosened further and allowed the W-block to sag more . . . resulting in a runaway failure. The entire beam became occluded and the upper block became so hot that the two steel M8 screws at the front evaporated, and the upper and lower W-blocks fused together. The absorbed power was over 8 kW. Note that the boiling point of iron is 3134 K, while the melting point of tungsten is 3695 K!
The parameter $K$

$$K = \alpha \gamma$$

Ratio between the wiggling angle of the trajectory $\alpha$, and the natural aperture of synchrotron radiation $1/\gamma$
The parameter $K$

For one electron moving in a sinusoidal magnetic field:

$$K = \frac{e}{2\pi m_0 c} \lambda_u B = 0.934 \lambda_u [\text{cm}] B [\text{T}]$$

In wigglers $K \gg 1$
• In an undulator $K \leq 1$, i.e. the wiggling angle $\alpha$ is smaller than, or close to, the photon natural emission angle $1/\gamma$

• The radiation emitted by the electrons at different points of the trajectory may interfere

• The interference filters out the photon wavelength/frequency for which the interference is not constructive.

• Therefore the emission becomes a narrow band around the a fundamental wavelength/frequency plus a series of higher harmonics
Undulator radiation is characterized by three parameters:

- The energy parameter $\gamma$
- The undulator spatial period $\lambda_u$
- The maximum angular deviation of the electron $\alpha$, and therefore $K$
Horizontal and vertical divergence are the same

\[ \sigma_h = \sigma_v \approx \frac{1}{\sqrt{nN}} \gamma \]

where \( N \): number of periods of the
\( n \): harmonic number

- For a typical undulator with 100 poles \( \sigma_h = \sigma_v \approx 10 \mu \text{rad} \)
- In practice \( \sigma_h \approx 100 \mu \text{rad} \) when considered the convolution with the electron beam divergence
The transformation from wiggler to undulator radiation is achieved in practice not by reducing the lateral excursion by decreasing the magnetic field strength between the magnetic pole pairs (drop in flux), but instead by reducing the magnetic pole periodicity $\lambda_u$

\[
K = \frac{e}{2\pi m_0 c} \lambda_u B = 0.934\lambda_u [\text{cm}] B [\text{T}]
\]

In undulators $K \sim 1$
Undulator equation - 1

\[ \lambda = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} + \theta^2\gamma^2 \right) \]

In the orbit plane

- Undulator period
- Wiggling angle
- Electron energy
- Observation angle
• In addition to the fundamental wavelength, also higher harmonics of shorter wavelength, $\lambda_n = \lambda/n$, are emitted.

• Their number and intensity increases with $K$;

• For $\theta = 0$:

$$\lambda = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$

• In practical units:

$$\lambda[\text{Å}] = \frac{13.056 \lambda_u[\text{cm}]}{n\varepsilon^2[\text{GeV}]} \left( 1 + \frac{K^2}{2} \right)$$
• In energy:

$$E_n[\text{keV}] = 0.95 \frac{n\varepsilon^2[\text{GeV}]}{(1+K^2/2)\lambda_u[\text{cm}]}$$

• The undulator spectrum consists of a set of narrow lines equally spaced in energy:

$$\Delta E = \frac{2hc\gamma^2}{\lambda_u(1 + K^2/2)}$$
Undulator equation implications - 1

\[ \lambda = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} + \theta^2\gamma^2 \right) \]

If we can change B we can change \( \lambda \)

For this reason, undulators are built with smoothly adjustable B field (by adjusting the gap between the two jaws of the undulators).
Undulator equation implications - 2

\[ \lambda = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} + \theta^2 \gamma^2 \right) \]

As B increases (and so K), the output wavelength increases (photon energy decreases).
This appear to be different to bending magnets and wigglers where to increase B is to produce shorter wavelengths (higher photon energies).

The wavelength changes with \( \theta^2 \), so it always gets longer as moving away from the axis.
This means that the beamline aperture choice is important because it alters the radiation characteristics reaching the observer.
Summary of the three basic sources

Bending magnet radiation

Wiggler radiation

Undulator radiation
It allows to compare the quality of the x-ray beams from different sources

Brilliance = \frac{\text{photons /second}}{(\text{mrad}^2)(\text{mm}^2 \text{ source area})(0.1\% \text{ bandwidth})}

![Graph showing the increase in brilliance over time from different generations of synchrotrons and wigglers.](image)
• Product of the linear source size and the beam divergence in the same plane
• The goal is to make this constant as small as possible
• There is a fundamental lower limit, given by Heisenberg’s uncertainty principle: $\epsilon_{\text{min}} = \frac{\lambda}{4\pi}$. For 1 Å, $\epsilon_{\text{min}} = 8 \text{ pm rad}$.
• In practice, the emittance is always higher than this, and this is a convolution of the emittance of the electron beam circulating within the storage ring, and that of the photon beam generated by a single electron passing through the source.
Emittance - 2

\[ \epsilon_x = \sigma_x \sigma'_x \]

\[ \epsilon_x = \sigma_y \sigma'_y \]

\(\sigma_x\) and \(\sigma_y\) are the standard deviation of Gaussians describing the beam profile in the \(x\)- and \(y\)- directions:

\[ I = I_0 e^{-x^2/2\sigma_x^2} e^{-y^2/2\sigma_y^2} \]