IAEA
International Atomic Energy Agency

LESSON 9:

MEASUREMENT AND UNCERTAINTY

CONTENT

- Introduction
- Measurements
- Uncertainty and errors
- Standard deviation and its calculation
- Sources of errors and types of errors
- Detection limits and Minimum Detectable Activity

Objectives

Provide a brief overview of basic concepts on measurements and uncertainty.

This lesson focuses on the measurement methods in counting the radioactive samples.

To provide the insights into standard deviation, detection limit and MDA.

Counting Statistics

IAEA

Radioactive decay is a random process. Consequently, any measurement based on observing the radiation emitted in nuclear decay is subject to statistical fluctuation

These inherent fluctuations represent an unavoidable source of uncertainty in all nuclear measurements.

The term counting statistics includes the framework of statistical analysis required to understand and interpret the results of a nuclear counting experiment.

Counting statistics is also used to make a prediction about the expected uncertainty of these measurements.

Measurement

What is a measurement?

- A measurement tells us about a property of something. It might tell us how heavy an object is, or how hot, or how long it is. A measurement gives a number to that property.

What is not a measurement?

- There are some processes that might seem to be measurements, but are not.
- For example, comparing two pieces of string to see which is longer is not really a measurement.

Uncertainty of Measurement

It tells something about its quality.

Uncertainty of measurement is the doubt that exists about the result of any measurement.

Expressing uncertainty of measurement

Two numbers are really needed in order to quantify an uncertainty. One is the width of the margin, or ***interval***. The other is a ***confidence level***, and states how sure we are that the ‘true value’ is within that margin.

Error versus Uncertainty

Error is the difference between the 'measured value' and the 'true value'.

Uncertainty is a quantification of the doubt about the measurement result.

Whenever possible we try to correct for any known ***errors***: for example, by applying ***corrections*** from calibration certificates. But any error whose value we do not know is a source of uncertainty.

Why is the Uncertainty of Measurement Important?

- You may be interested in uncertainty of measurement simply because you wish to make good quality measurements and to understand the results.

- You may be making the measurements as part of a:
  - **calibration** - where the uncertainty of measurement must be reported on the certificate.
  - **test** - where the uncertainty of measurement is needed to determine a pass or fail.
  - **tolerance** - where you need to know the uncertainty before you can decide whether the tolerance is met.

Spread…standard deviation *(σ)*

The standard deviation is a measure of the dispersion of randomly occurring events around a mean.

The usual way to quantify spread is *standard deviation*. The standard deviation of a set of numbers tells us about how different the individual readings typically are from the average of the set.

As a ‘rule of thumb’, roughly two thirds of all readings will fall between plus and minus (±) one standard deviation of the average. Roughly 95% of all readings will fall within two standard deviations. This ‘rule’ applies widely although it is by no means universal.

Calculating an estimated standard deviation *(σ)*

Consider the set of readings 1,2,3,4,5,6,7 and the average is 4.

→

Next, find the difference between each reading and the average and it comes -3,-2.-1,0,1,2,3

→

Next square each of these, i.e. 6, 4, 1, 0, 1, 4, 6.

→

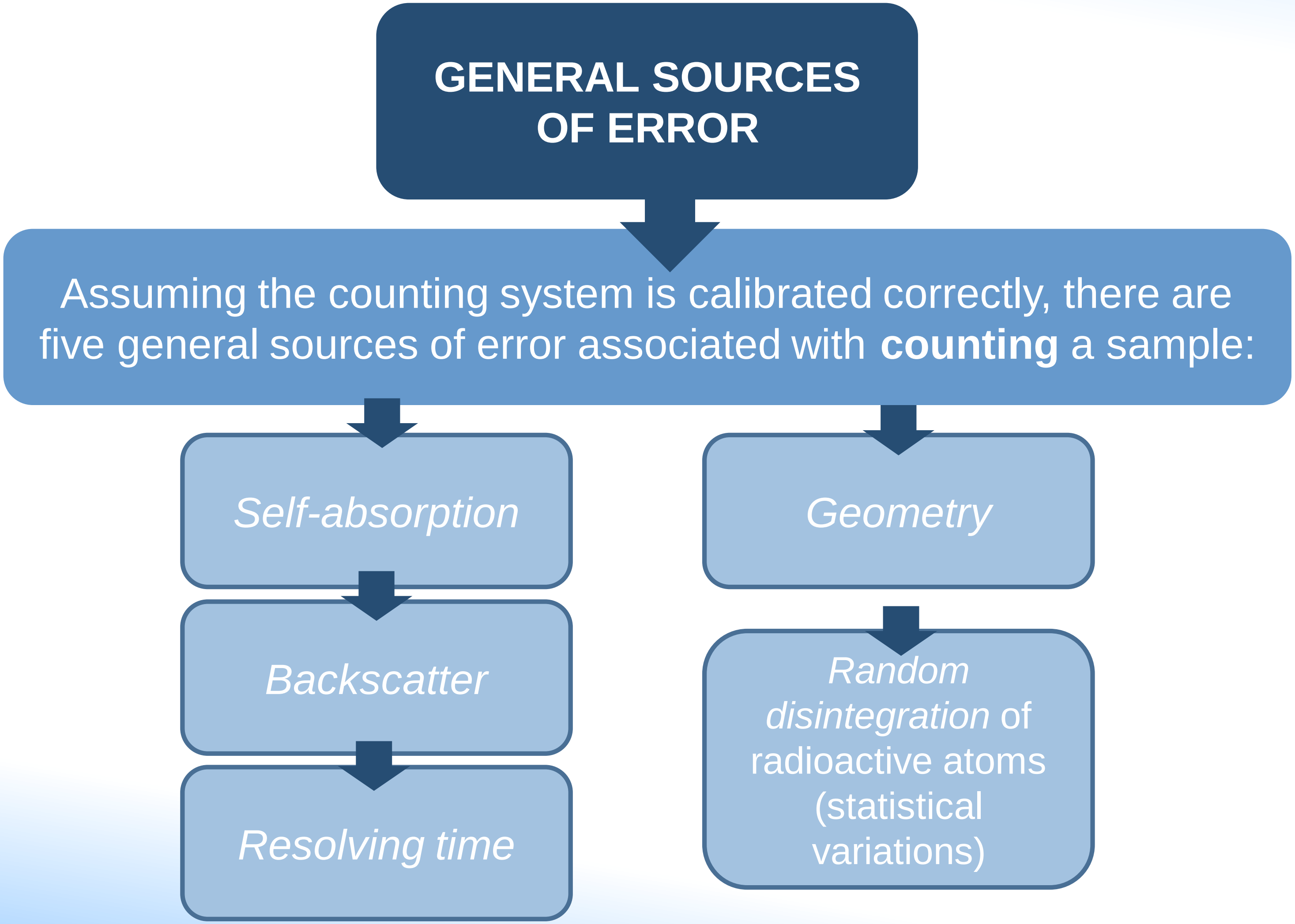
Next, find the total and divide by *n*-1, (in this case *n* is 7, so *n*-1 is 6),

i.e. $\frac{6+4+1+0+1+4+6}{6} = 3.66$

→

The estimated standard deviation (*σ*) is found by taking the square root of the total, $\sigma=\sqrt{3.66} = 1.91$.

Sources of Errors and Uncertainty



Types of Errors

- Systematic Errors
  - uncertainties in the bias of the data, such as an unknown constant offset, instrument mis-calibration
  - implies that all measurements are shifted the same (but unknown) amount from the truth.
  - measurements with a low level of systematic error, or bias, have a high accuracy.

- Random Errors
  - arise from inherent instrument limitation (e.g. electronic noise) and/or the inherent nature of the phenomena (e.g. biological variability, counting statistics)
  - each measurement fluctuates independently of previous measurements, i.e. no constant offset or bias – measurements with a low level of random error have a high precision.

Probability Distribution Function

These functions, characterize random errors with a distribution Statistical Models:

1. Binomial Distribution Random independent processes with two possible outcomes.

2. Poisson Distribution Simplification of binomial distribution with certain constraints.

3. Gaussian or Normal Distribution Further simplification if average number of successes is large (e.g. >20).

Simple Examples

The following are measurements of counts per minute from a ^{22}Na source. What is the count rate and its uncertainty?

- 2204, 2210, 2222, 2105, 2301
- x = 2208.4 hence $\sigma = \sqrt{2208.4/5} = 21$
- Therefore count Rate = (2208 ± 21)counts/min

The Effect of Background (1)

!

- When the measurement result includes a contribution from the radiation background, the background value must be subtracted from the measured value.

!

- Results of repeated background measurements are also random and follow a normal distribution

The Effect of Background (2)

!

- The difference between sample and background values also follows a normal distribution with true mean equal to the difference of true sample and background means, but with variability greater than the variability of sample or background counts alone

!

- If the counting times of sample and background are equal, then the best estimate of the standard deviation of the difference is given by the square root of the sum of the two measurements, except when they consist of only a few counts

$$s = \sqrt{N + B}$$

The Effect of Background (3)

- ❑ Background count (600 s) = 380 counts
- ❑ Sample count (600 s) = 650 counts
- ❑ Net counts = 650 – 380 = 270 counts

  - Standard deviation of the difference, $\sigma = \sqrt{(650 + 380)}$ = 32.1 counts
  - 95 % conf. level (1.96 σ) = 62.9 counts
  - The net count may be expressed as 270 ± 63, at 95 % confidence level
  - To get the count rate, divide by 600 s: 0.45 ± 0.11 cps

LC, LD and MDA (1)

* LC = critical level
* LD = limit of detection
* MDA = minimum detectable concentration

* The critical level (LC) is the level, in counts, at which there is a predetermined statistical probability of incorrectly identifying a measurement system background value as “greater than background”

* Any response above the LC is considered to be greater than background

IAEA

LC, LD and MDA (2)

IAEA

- The limit of detection (LD) is an *a priori* estimate of the detection capability of a measurement system, and is also reported in units of counts.
- The minimum detectable concentration (MDA) is the LD in counts multiplied by an appropriate conversion factor to give units consistent with a required measurement, such as Bq/cm^2 or Bq/m^3.
- MDA is the lowest activity value that can be achieved when a sample is measured with a detection system.

LC, LD and MDA (3)

- Gamma-ray spectrometry: MDA depends on the background (statistical nature), counting time, detector and sample properties, measurement geometry, nuclear 2 and sample properties, measurement geometry, nuclear decay data of the considered radionuclide.

- The MDA depends on the
  - Background of counting system
  - Counting efficiency of a counting system
  - Counting time

LC, LD and MDA (4)

The critical level and the detection limit can be calculated by using the following formulae:

$$L_C = k\sqrt{2B} \qquad L_D = k^2 + 2k\sqrt{2B}$$

where

L_C = critical level (counts)

L_D = detection limit (counts)

k = corresponds to (1-α) and (1-β) probability levels of the standardized normal distribution (assuming α and β are equal)

B = number of background counts that are expected to occur while performing an actual measurement.

LC, LD and MDA (4)

If values of 0.05 for both α and β are considered acceptable, then k = 1.645 (from look-up tables) and the equations can be written as:

$$L_C = 1.65\sqrt{B} \qquad L_D = 2.71 + 3.29\sqrt{B}$$

$$MDA = confidence.level\ (2.71 + 3.29\sqrt{B}) / T * E$$

Where C is the conversion factor

MDA for Air Sampling Systems*

$$MDA = \frac{2.71 + 3.29\sqrt{R_b\, T_s\, (1 + T_s / T_b)}}{E\, F\, K\, T_s\, T_b}$$

R_s = sample count rate in s^{-1}
T_s = sample counting time in s
T_b = background counting time in s
E = fractional filter efficiency in (%)
F = air flow rate through sampler in $m^3\ s^{-1}$
K = counting efficiency in $s^{-1}\ Bq^{-1}$

*NUREG 1400

Conclusions

- Statistics is a part of measuring
- Counting values have uncertainties
- Detection level and MDA are important parameters helps in reporting the results confidently.